University of Richmond
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The CAPM and Multifactor Pricing Models

Fin 366: Investments

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THE CAPITAL ASSET PRICING MODEL (CAPM)

Given the index model,

$$R_{i,t} = \alpha_i + \beta R_{M,t} + e_{i,t}$$

good portfolio managers and investors should:

1. *Buy positive alpha stocks* → positive return beyond what is explained by the model and non-diversifiable market risk

2. *Sell short or avoid negative alpha stocks* → negative return beyond what is explained by the model and non-diversifiable market risk

Thus, we might expect that:

Investors flock to buy positive alpha → bid up price, reducing expected return, eliminating alpha.

> **EXAMPLE**: Shares of Websturco Inc. trade at $100, but "should" be trading at $200 given the company's characteristics.
>
> - Investor A buys today at $100
> - Investor B waits and buys later when it trades at $150
> - Investor C waits and buys later when it trades at $199
> - All sell once Websturco shares reach $200, yielding a 100% gain for A, a 33.3% gain for B, and a 0.5% gain for C.

Investors avoid or sell negative alpha → reduction in price, increases expected return, pushes up alpha toward zero.

Investors can easily identify and compute asset betas and alphas. In a world of millions of traders, we expect that in theory:

Buying and selling pressures will leave most securities with zero alphas most of the time.

IMPLICATION 1: When alpha is zero, there is no reward for bearing unsystematic (firm-specific) risk

IMPLICATION 2: The risk premium (return in excess of the risk-free rate) will be determined *solely* by bearing systematic risk as measured by beta

IMPLICATION 3: With no reward for diversifiable/unsystematic/firm-specific risk, the best portfolio is the one that completely eliminates it – an index fund

And these three implications result in the **Capital Asset Pricing Model** (CAPM).

The **CAPM** relates the required rate of return on a security *i* to its systematic risk, as measured by beta β:

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$

where

$E(r_i)$	Expected return on security *i*
r_f	Risk-free rate, proxied by the return on 90-day T-Bills
β_i	Beta, the measure of systematic risk of the security, *estimated* by the index model regression
$E(r_M)$	Expected return of the market
$[E(r_M) - r_f]$	**Market risk premium**, not to be confused with the *expected return of the market*

Note the clear similarities between the index model and the CAPM. The CAPM is essentially a theoretical *equilibrium*, whereby the α of the index model is zero by the discussion above.

By the model, we can compute the expected or required rate of return on a security given CAPM inputs.

EXAMPLE: What is the expected return on Disney stock assuming the arithmetic average of the S&P 500 over the last 20 years is 12%, the annualized return on 90-day T-bills is 1%, and historically, Disney rises 1.2% for every 1% increase in the market?

EXAMPLE: Suppose Disney stock actually returned 16.20% despite your expectations. What is Disney's alpha, and why might this stock have alpha when the CAPM implies it should not?

EXAMPLE: Estimate beta of a portfolio of stocks if the expected return on the portfolio is 18%, T-bills yield 1%, and the Wilshire 5000's expected return is 14%.

The Security Market Line: A Graphical Depiction of the CAPM

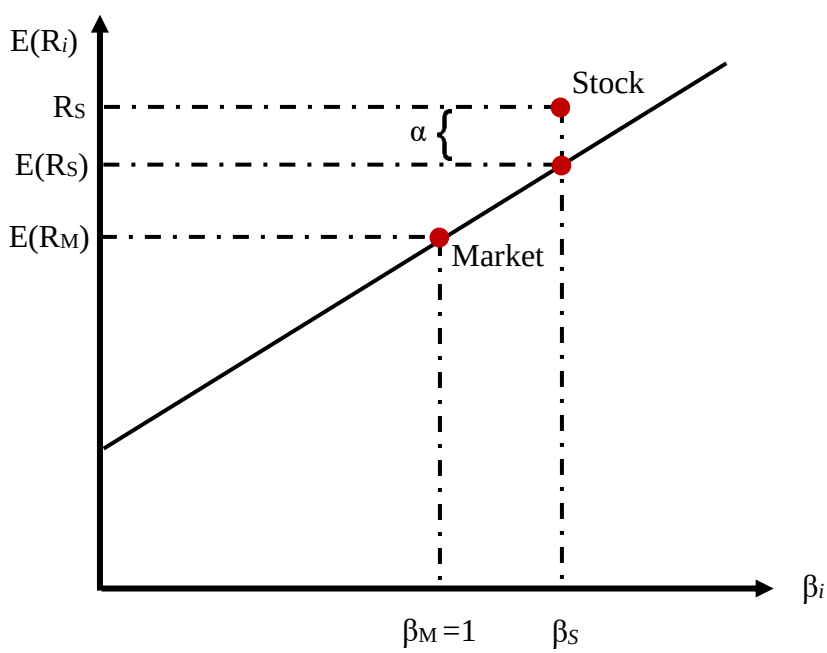
The **Security Market Line (SML)** is the graphical representation of the relationship between expected return and beta of the CAPM.

The y-axis is the reward $E(r_i)$ while the x-axis is the risk β_i. Note how this differs from the CAL, CML, and SCL.

> ? What are the axes for the CAL and CML? What about the SCL?

An appropriate measure of risk for a portfolio is σ, but the relevant measure of risk for an individual asset (when part of a diversified portfolio) is β. Given the CAPM is the linear relationship between the E(R) of an asset and the market portfolio, we can depict it as a line.

Figure 1: The Security Market Line



- Stocks falling *above* the line are *undervalued*. That is, they offer some positive alpha because their return is greater than the expected return by the model.

- Stocks falling *below* the line are *overvalued*. That is, they offer some negative alpha because their return is less than the expected return by the model.

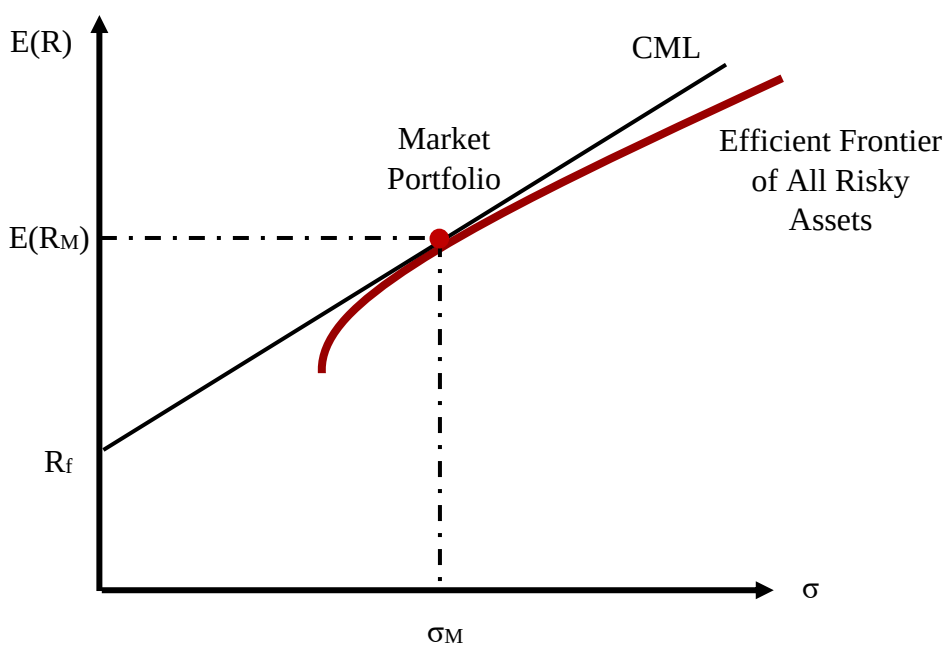
The Excel file *CAPM and SML* available at josephfarizo.com/fin366.html provides additional examples, practice problems, and illustrations.

CAPM Implications

By the CAPM, all investors desire the same value-weighted market portfolio of all assets.

A PRIMARY IMPLICATION OF THE CAPM IS THAT THE PASSIVE STRATEGY IS EFFICIENT: THE *MARKET PORTFOLIO* WILL BE ON THE EFFICIENT FRONTIER AND WILL BE THE OPTIMAL RISKY PORTFOLIO.

Figure 2: The Market Portfolio as the Optimal Risky Portfolio



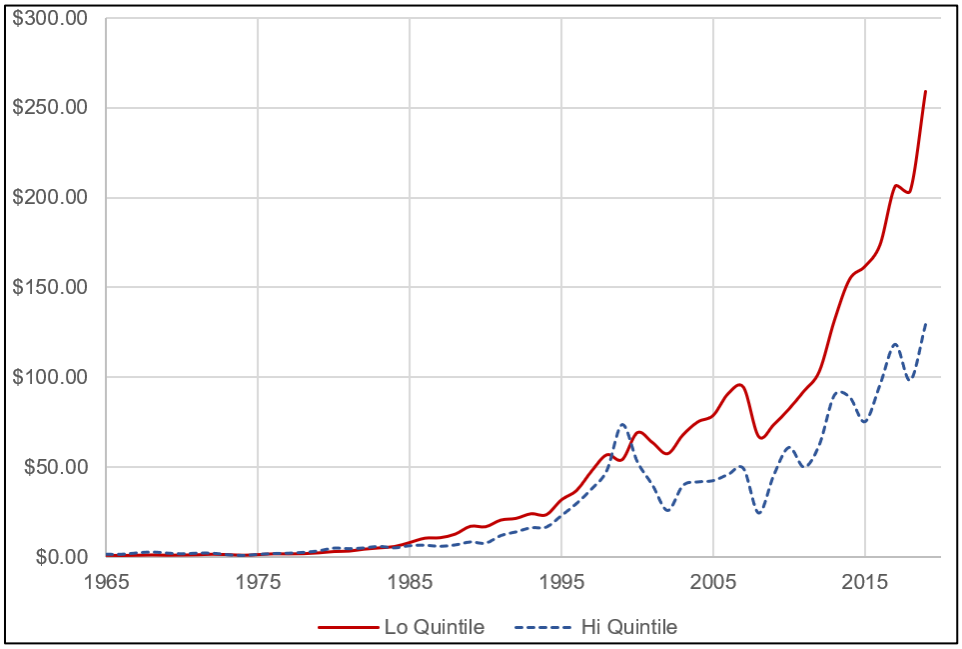
CAPM in Practice

The CAPM relies on so many assumptions that it fails many empirical tests. It requires that all investors have homogenous expectations and access to the same information. It assumes all assets are publicly tradable, and that there are no taxes or transaction costs. This is how we arrive at the result that asset alphas are zero most of the time.

> *I can summarize decades worth of academic research in two sentences: At best, the empirical evidence is inconclusive about whether the CAPM should be discarded. At worst, it is conclusive and the CAPM should be completely discarded.*
>
> Ivo Welch
> Distinguished Professor of Finance
> UCLA Anderson School of Management

By the CAPM, a stock's expected return should be higher if its beta is higher. Is this true?

Figure 3: Growth of $1 for Low and High Beta Stocks



See the Excel file *CAPM Performance* available at josephfarizo.com/fin366.html for the data used to generate the previous figure.

Given its weaknesses, why the CAPM?

- ACROSS VS. WITHIN ASSET CLASSES: Indeed, stocks have higher average returns than bonds. *Across* asset classes rather than *within*, CAPM performance is better.

- INTUITION AND DIVERSIFICATION: Be aware of systematic risk and diversify away from unsystematic risk.

- MODEL SIMPLICITY: Only a few inputs are required. Related to the point above, you should not be rewarded for diversifiable risk.

- WIDELY USED: some 72% of valuation professionals always or almost always use it.[1] Alternatives may not be as simple or viable.

- A STARTING POINT FOR QUANTIFYING RISK: While it may not characterize returns well, it may guide managers on exposure to market risk.

MULTIFACTOR PRICING MODELS

In reality, systematic risk comes from a number of economywide factors, for example business-cycle risk, inflation, and energy prices. We can think about allowing stocks to have different *sensitivities* to different components of systematic risk.

THE FAMA-FRENCH 3 FACTOR MODEL

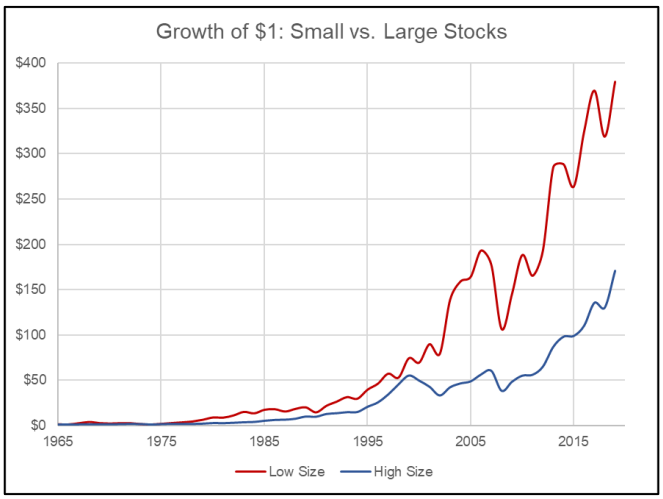
Eugene Fama (Nobel Prize Winner) and Ken French find that firm size and book-to-market ratios capture risk that the CAPM beta doesn't capture.

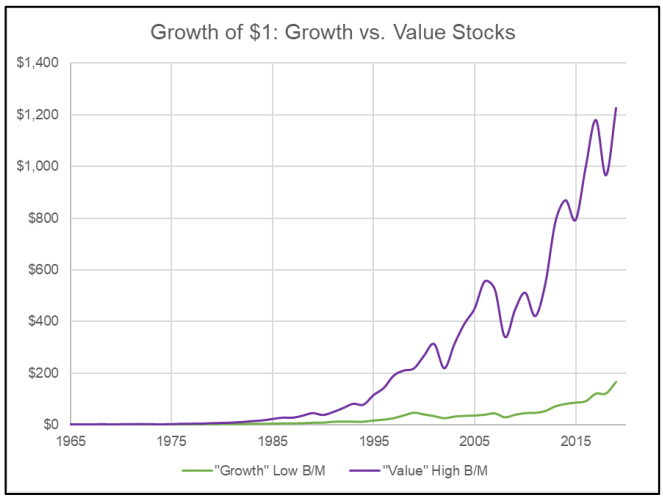
1. **Size**: smaller firms have historically outperformed larger firms

2. **Book-to-Market**: value stocks have historically outperformed growth stocks

> ℹ Recall **value stocks** have a high book value of equity relative to market value of equity (↑B/↓M) while **growth stocks** have a low book value of equity relative to market value of equity (↓B/↑M).

Figure 4: The Size and Value Effects





See the Excel file *Size and Value Performance* available at josephfarizo.com/fin366.html for the data used to generate the previous figure.

The CAPM can be supplemented with factors relating to the size and value effects that Fama & French uncover in the data. That is, the CAPM formula

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$

becomes

$$E(r_i) = r_f + \beta_{i,M}[E(r_M) - r_f] + \beta_{i,SMB}\underbrace{E(r_{SMB})}_{\text{E(R) of small stocks minus large stocks (Small Minus Big)}} + \beta_{i,HML}\underbrace{E(r_{HML})}_{\text{E(R) of high B/M stocks minus low B/M stocks (High Minus Low)}}$$

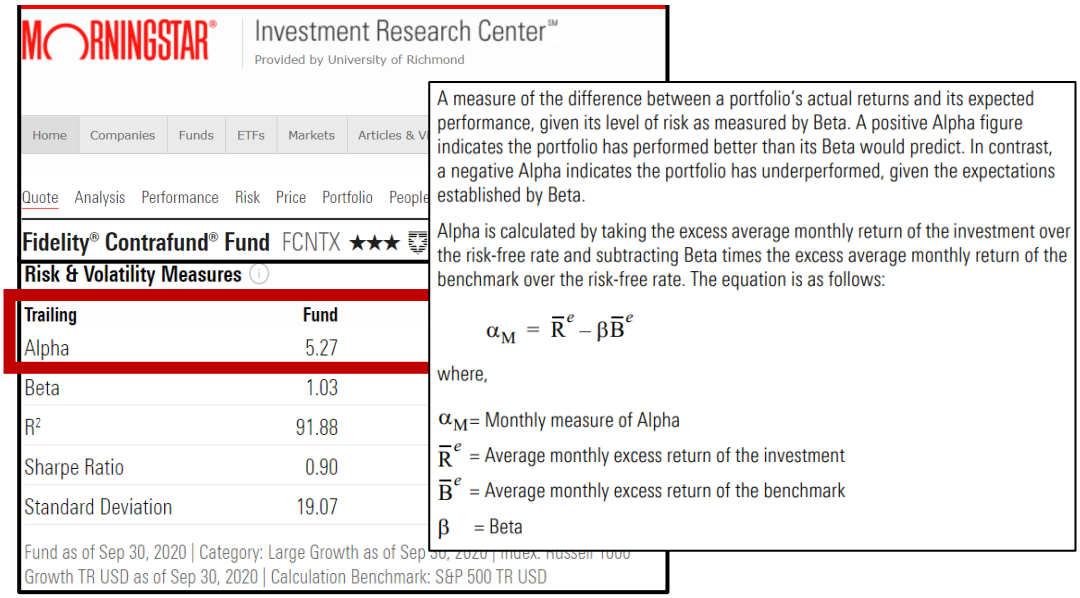
where

$\beta_{i,M}$	Sensitivity to the market factor
$\beta_{i,SMB}$	Sensitivity to the size factor
$\beta_{i,HML}$	Sensitivity to the value factor

Like the index model or the CAPM, beta estimates can be obtained via regression. The factors themselves are the differences in returns between the two types of stocks. Generally, we expect the R^2 of the model to increase relative to the CAPM as we account for additional risk factors.

Additionally, we often expect that alphas in the Fama-French model will be *attenuated*, or nearer to zero, for a security relative to the CAPM. What might have appeared as risk-adjusted returns by the CAPM may be "picked up" by the additional risk factors in the Fama-French 3 Factor Model.

Figure 5: Morningstar Alphas



Which alphas, CAPM or Fama-French, does Morningstar[2] present? How should we be careful when interpreting these alphas?

EXAMPLE: What's the expected return on FLSL Corp. stock? The market risk premium is 11%. The current SMB and HML factors are 2% and 3%. When you regressed the return of FLSL on the SMB and HML factors, you found the stock's sensitivity to the market risk premium was 1.2 while the sensitivities to the size and value components of systematic risk are 1.4 and -1.3. Alpha is 1%. The risk-free rate is 1%. (*Hint*: what do we do with alpha for expected returns?)

Additional Factors

Why the size and value factors? These factors are incorporated because Fama and French revealed the differences in returns between small/large and value/growth stocks we've discussed before. Given these differences in returns, there must be *some fundamental risk source* associated with small stocks and value stocks.

Thus, these factors are *proxies for fundamental risk sources* not fully captured by the CAPM's market beta. Other factors may supplement the model further, accounting for **anomalies** in stock returns.

$$E(r_i) = r_f + \beta_{i,M}\left[E(r_M) - r_f\right] + \beta_{i,SMB}E(r_{SMB}) + \beta_{i,HML}E(r_{HML}) + \beta_{i,UMD}E(r_{UMD}) + \beta_{i,CMA}E(r_{CMA}) + \beta_{i,RMW}E(r_{RMW})$$

where

UMD represents **momentum**, or "up minus down"

CMA represents **investment**, or "conservative minus aggressive"

RMW represents **profitability**, or "robust minus weak"

and the betas are "**factor loadings**" representing exposures to those risk sources, or factors.

? A security has a positive beta on the market risk premium, UMD, CMA, RMW, and SMB factors but a negative beta on the HML factor. What does this tell us about this security's expected return and *exposure* to these risk sources?

Ken French's website[3] and the Wharton Research Data Services (WRDS) website[4] include a number of historical risk factors that may be used in regressions to determine betas.

END NOTES

[1] 72% of valuation professionals always or almost always use the CAPM: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2784850
[2] You can obtain mutual fund alphas from Morningstar at https://library-morningstar-com.newman.richmond.edu/mirc/home.aspx
[3] Ken French's website: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[4] WRDS: https://wrds-www.wharton.upenn.edu/