UNIVERSITY OF RICHMOND
Robins School of Business™

§11. CAPM AND MULTIFACTOR PRICING MODELS

FIN 366: INVESTMENTS

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THE INDEX MODEL

Recall the formula and inputs used to determine the optimal weights of assets in the two-risky asset scenario. To construct an efficient frontier from 50 securities, you'd have to make 1,325 calculations:

- 50 expected returns for the 50 securities
- 50 standard deviations for the 50 securities
- 1,225 covariances and correlations among the securities.

For 3,000 securities, you'd need more than 4.5 *million* estimates.[1] Obviously, this is a computationally intense problem handled by software. Rather than produce the millions of computations required for expected returns, variances, and covariances to develop an optimal risky portfolio, we can recognize that positive covariates among security returns arise from common economic forces that affect the prospects of most firms.

The **Index Model** is a model relating stock returns to returns on both a broad market (systematic) and firm-specific (unsystematic) factors. We take the *overall market excess return* to be this index and determine security "exposures" or "sensitivities" to this index.

We begin by graphically depicting the index model:

1. Plot an asset's excess returns (y-axis) relative to the market index's excess return (x-axis), where excess return is the return in excess of the risk-free rate.

2. Determine the line of best fit (via regression) through the plot to describe the *typical* relationship between the return on the security and the return on the market.

    - Slope: the security's *sensitivity* to the market return (**beta** or β)

    - y-intercept: the security's return when the market return is zero (**alpha** or α)

    - R^2: the portion of security's variation explained by the market

    - $(1-R^2)$: the portion of the security's risk that is firm specific

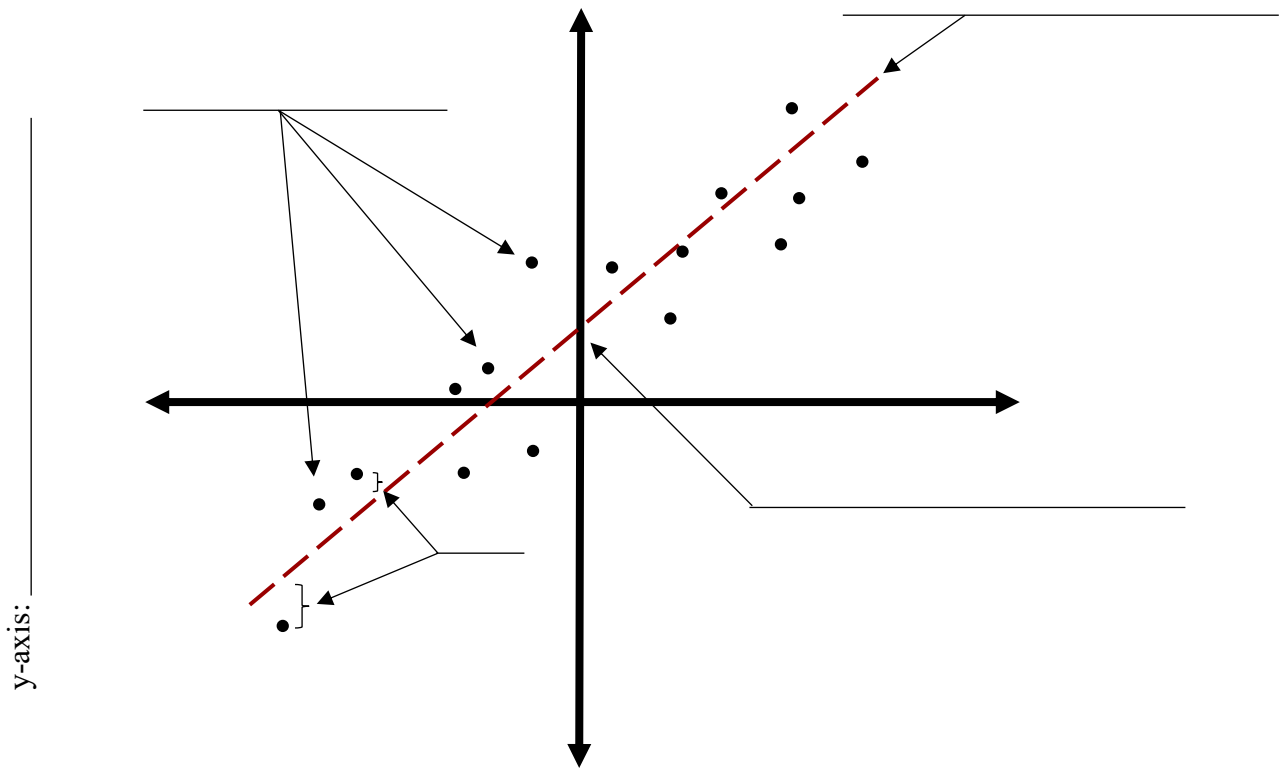
Given this relationship, the index model can be written as the following *regression equation*, following the standard $y = mx + b$ format. We regress the excess returns of the market on the excess returns of a security:

$$R_{i,t} = \alpha_i + \beta R_{M,t} + e_{i,t}$$

where the excess return *R* on security *i* in time *t* is linearly related to the excess return on the market *M* at time *t*. The error term *e* represents the firm-specific "surprise" in the security return in that time *t*.

The **Security Characteristic Line** is the line drawn through the scatterplot, with the above equation.

For further illustration, we will use the Excel file *Index Model* available at josephfarizo.com/fin366.html. Below are axes for you to draw on and label.



y-axis: _____

x-axis: _____

[4]

Can beta be negative? What about alpha or R-squared?

THE CAPITAL ASSET PRICING MODEL

Given the index model

$$R_{i,t} = \alpha_i + \beta R_{M,t} + e_{i,t}$$

good portfolio managers and investors should:

1. *Buy positive alpha stocks* → positive return beyond what is explained by the model and non-diversifiable market risk

2. *Sell short or avoid negative alpha stocks* → negative return beyond what is explained by the model and non-diversifiable market risk

Thus, we might expect that:

Investors flock to buy positive alpha → bid up price, reducing expected return, eliminating alpha.

Investors avoid or sell negative alpha → reduction in price, increases expected return, pushes up alpha toward zero.

EXAMPLE: Shares of Websturco Inc. trade at $100, but "should" be trading at $200 given the company's characteristics.

- Investor A buys today at $100
- Investor B waits and buys later when it trades at $150
- Investor C waits and buys later when it trades at $199
- All sell once Websturco shares reach $200, yielding a 100% gain for A, a 33.3% gain for B, and a 0.5% gain for C.

Investors can easily identify and compute asset betas and alphas. In a world of millions of traders, we expect that in theory:

Buying and selling pressures will leave most securities with zero alphas most of the time.

IMPLICATION 1: When alpha is zero, there is no reward for bearing unsystematic (firm-specific) risk.

IMPLICATION 2: The risk premium (return in excess of the risk-free rate) will be determined *solely* by bearing systematic risk as measured by beta.

IMPLICATION 3: With no reward for diversifiable/unsystematic/firm-specific risk, the best portfolio is the one that completely eliminates it – an index fund.

And these three implications result in the **Capital Asset Pricing Model** (CAPM).

The **CAPM** relates the required rate of return on a security *i* to its systematic risk, as measured by beta β:

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$

where

$E(r_i)$	Expected return on security *i*
r_f	Risk-free rate, proxied by the return on 90-day T-Bills
β_i	Beta, the measure of systematic risk of the security, *estimated* by the index model regression
$E(r_M)$	Expected return of the market
$[E(r_M) - r_f]$	**Market risk premium**, not to be confused with the *expected return of the market*

Additionally, beta may be found by dividing the covariance of the market and the security *i* returns by the variance of the market returns, or:

$$\beta_i = \frac{COV(r_i, r_M)}{\sigma_M^2} = \frac{\rho_{i,M}\sigma_i\sigma_M}{\sigma_M^2} = \frac{\rho_{i,M}\sigma_i}{\sigma_M}$$

Note the clear similarities between the index model and the CAPM. The CAPM is essentially a theoretical *equilibrium*, whereby the α of the index model is zero by the discussion above.

By the model, we can compute the **expected** or **required rate of return** on a security given CAPM inputs.

PRACTICE: What is the expected return on Disney stock assuming the arithmetic average of the S&P 500 over the last 20 years is 12%, the annualized return on 90-day T-bills is 1%, and historically, Disney rises 1.2% for every 1% increase in the market?

PRACTICE: Suppose Disney stock actually returned 16.20% despite your expectations. What is Disney's alpha, and why might this stock have alpha when the CAPM implies it should not?

PRACTICE: Estimate beta of a portfolio of stocks if the expected return on the portfolio is 18%, T-bills yield 1%, and the Wilshire 5000's expected return is 14%.

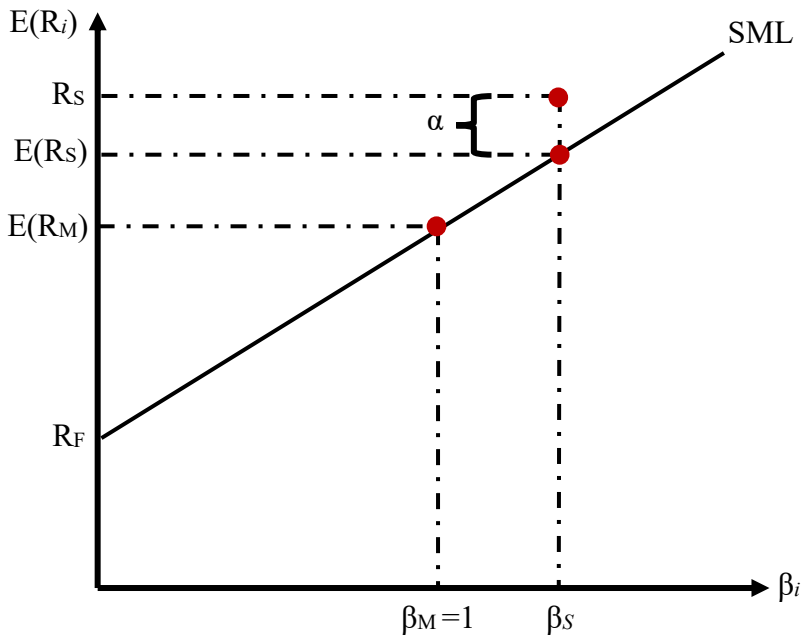
THE SECURITY MARKET LINE

The **Security Market Line (SML)** is the graphical representation of the relationship between expected return and beta of the CAPM.

The y-axis is the reward $E(r_i)$, while the x-axis is the risk β_i. Note how this differs from the CAL, CML, and SCL.

An appropriate measure of risk for a portfolio is σ, but the relevant measure of risk for an individual asset (when part of a diversified portfolio) is β. Given the CAPM is the linear relationship between the E(R) of an asset and the market portfolio, we can depict it as a line.

Figure 1: The Security Market Line



- Securities falling *above* the line are *undervalued*. That is, they offer some positive alpha, or **risk-adjusted return**, because their return is greater than the expected return by the model.

- Securities falling *below* the line are *overvalued*. That is, they offer some negative alpha, or **risk-adjusted return**, because their return is less than the expected return by the model.

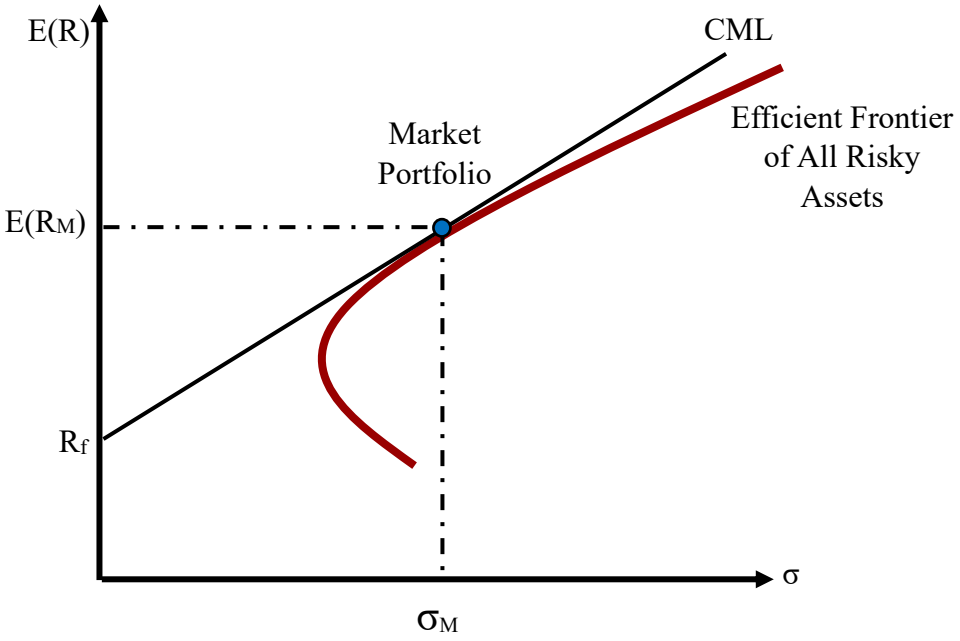
The Excel file *CAPM and SML* available at josephfarizo.com/fin366.html provides additional examples, practice problems, and illustrations

CAPM Implications

By the CAPM, all investors desire the same value-weighted market portfolio of all assets.

A primary implication of the CAPM is that the passive strategy is efficient: the *market portfolio* will be on the efficient frontier and will be the **optimal risky** portfolio.

Figure 2: The Market Portfolio as the Optimal Risky Portfolio



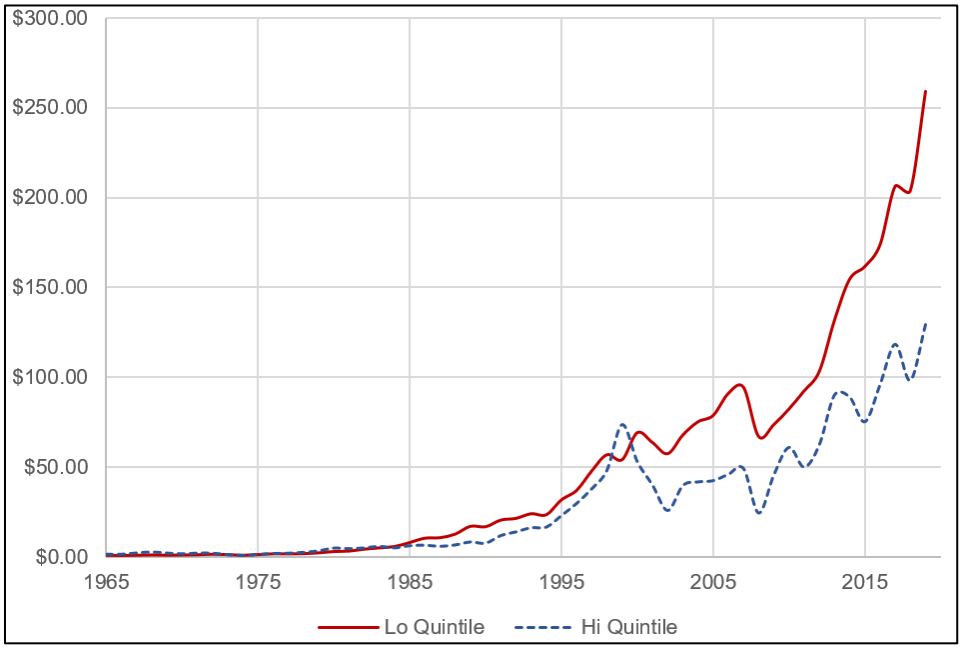
THE CAPM IN PRACTICE

The CAPM relies on so many assumptions that it fails many empirical tests. It requires that all investors have homogenous expectations and access to the same information. It assumes all assets are publicly tradable, and that there are no taxes or transaction costs. This is how we arrive at the result that asset alphas are zero most of the time.

> *I can summarize decades worth of academic research in two sentences: At best, the empirical evidence is inconclusive about whether the CAPM should be discarded. At worst, it is conclusive and the CAPM should be completely discarded.*
>
> Ivo Welch
> Distinguished Professor of Finance
> UCLA Anderson School of Management

By the CAPM, a stock's expected return should be higher if its beta is higher. Is this true?

Figure 3: Growth of $1 for Low and High Beta Stocks



See the Excel file *CAPM Performance* available at josephfarizo.com/fin366.html for the data used to generate the previous figure.

Given its weaknesses, why the CAPM?

- **ACROSS VS. WITHIN ASSET CLASSES**: Indeed, stocks have higher average returns than bonds. *Across* asset classes rather than *within*, CAPM performance is better.

- **INTUITION AND DIVERSIFICATION**: Be aware of systematic risk and diversify away from unsystematic risk.

- **MODEL SIMPLICITY**: Only a few inputs are required. Related to the point above, you should not be rewarded for diversifiable risk.

- **Widely Used**: some 72% of valuation professionals always or almost always use it.[2] Alternatives may not be as simple or viable.

- **A Starting Point for Quantifying Risk**: While it may not characterize returns well, it may guide managers on exposure to market risk.

Multifactor Pricing Models

In reality, systematic risk comes from a number of economywide factors, for example business-cycle risk, inflation, and energy prices. We can think about allowing stocks to have different *sensitivities* to different components of systematic risk.

The Fama-French 3 Factor Model

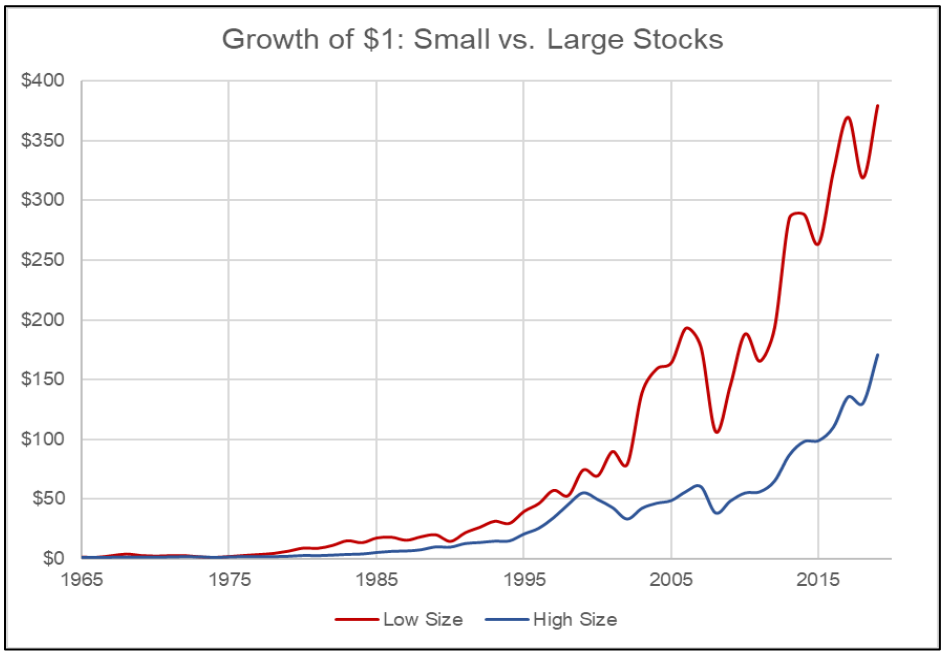
Eugene Fama (Nobel Prize Winner) and Ken French find that firm size and book-to-market ratios capture risk that the CAPM beta does not capture.

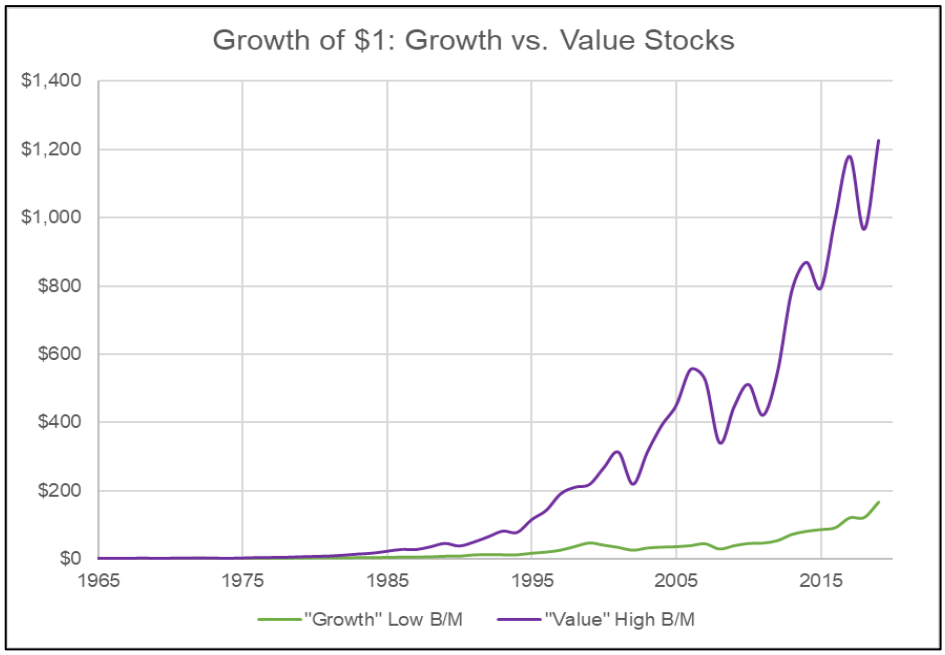
1. **Size**: smaller firms have historically outperformed larger firms.

2. **Book-to-Market**: value stocks have historically outperformed growth stocks.

> Recall **value stocks** have a high book value of equity relative to market value of equity (↑B/↓M) while **growth stocks** have a low book value of equity relative to market value of equity (↓B/↑M).

Figure 4: The Size and Value Effects





See the Excel file *Size and Value Performance* at josephfarizo.com/fin366.html for the data used to generate the previous figures.

The CAPM can be supplemented with factors relating to the size and value effects that Fama and French uncover in the data. That is, the CAPM formula

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$

becomes

$$E(r_i) = r_f + \beta_{i,M}\left[E(r_M) - r_f\right] + \beta_{i,SMB}E(r_{SMB}) + \beta_{i,HML}E(r_{HML})$$

E(R) of small stocks minus large stocks (Small Minus Big)

E(R) of high B/M stocks minus low B/M stocks (High Minus Low)

where

$\beta_{i,M}$	Sensitivity to the market factor
$\beta_{i,SMB}$	Sensitivity to the size factor
$\beta_{i,HML}$	Sensitivity to the value factor

These betas are **factor loadings** or **coefficients** in the regression representing exposures to those risk sources, or factors.

Like the index model or the CAPM, beta estimates can be obtained via regression. The factors themselves are the differences in returns between the two types of stocks. Generally, we expect the R^2 of the model to increase relative to the CAPM as we account for additional risk factors.

Additionally, we often expect that alphas in the Fama-French model will be *attenuated*, or nearer to zero, for a security relative to the CAPM. What might have appeared as risk-adjusted returns by the CAPM may be "picked up" by the additional risk factors in the Fama-French 3 Factor Model.

PRACTICE: What's the expected return on FLSL Corp. stock? The market risk premium is 11%. The current SMB and HML factors are 2% and 3%. When you regressed the return of FLSL on the SMB and HML factors, you found the stock's sensitivity to the market risk premium was 1.2 while the sensitivities to the size and value components of systematic risk are 1.4 and -1.3. Alpha is 1%. The risk-free rate is 1%. (*Hint*: what do we do with alpha for expected returns?)

Expanding the Fama French Model with Additional Factors

Why the size and value factors? These factors are incorporated because Fama and French revealed the differences in returns between small/large and value/growth stocks we've discussed before. Given these differences in returns, there must be *some fundamental risk source* associated with small stocks and value stocks.

Thus, these factors are *proxies for fundamental risk sources* not fully captured by the CAPM's market beta. Other factors may supplement the model further, accounting for **anomalies** in stock returns.

$$E(r_i) = r_f + \beta_{i,M}\left[E(r_M) - r_f\right] + \beta_{i,SMB}E(r_{SMB}) + \beta_{i,HML}E(r_{HML}) + \beta_{i,UMD}E(r_{UMD}) + \beta_{i,CMA}E(r_{CMA}) + \beta_{i,RMW}E(r_{RMW})$$

where

UMD represents **momentum**, or "up minus down"

CMA represents **investment**, or "conservative minus aggressive"

RMW represents **profitability**, or "robust minus weak"

Visit the link *Python Code: Multifactor Model Regressions* available at josephfarizo.com/fin366.html, enter a stock or security ticker of your choice, and execute the code to run multifactor model regressions.

Ken French's website[3] and the Wharton Research Data Services (WRDS) website[4] include a number of historical risk factors that may be used in regressions to determine betas. Additional factors can be related to other sources of risk, including **liquidity** and **volatility**.

An ETF has a positive beta (or coefficients/factor loadings) on the market risk premium, UMD, CMA, RMW, and SMB factors but a negative beta on the HML factor. What does this tell us about this security's expected return and *exposure* to these risk sources?

CRITICAL THINKING QUESTIONS

1. In what market conditions would you want a high beta stock? In what market conditions would you want a low beta stock?
2. In what market conditions would you want a high alpha stock?
3. What should the approximate beta of the inverse ETF ProShares Short S&P 500 ETF (ticker = SH) be? (Hint: You can check your answer by looking up ticker SH on Yahoo! Finance).
4. How is the security characteristic line obtained? How does it differ from the CAL and CML?
5. How does one determine if a stock has a positive or negative alpha?
6. Explain how the index model and the CAPM are related. Why is there no alpha in the CAPM?
7. Often, investment professionals calculate "CAPM betas" and "CAPM alphas" of securities by running regressions of a stock's excess return on the market's excess return. Explain why these are technically "index model betas and alphas".
8. How (in theory) are alphas eliminated when investors buy and sell positive and negative alpha stocks?
9. An implication of zero alphas means there is no reward for unsystematic risk. What source of risk offers reward? Why?
10. What is the market risk premium? How does it differ from the expected return of the market?
11. By the CAPM, what do we need in order to compute the expected return on a stock?
12. What is the formula for the expected return of a stock by the CAPM?
13. How do we estimate the expected return for the market?
14. Can you draw and label the SML?
15. Where would you find under- and over-valued stocks on the SML graph?
16. What is the value of the "beta of the market portfolio" and why is it always the same?
17. What is included in the market portfolio for the CAPM?
18. What is the optimal risky portfolio by the CAPM? Is it a passive or active portfolio?
19. Why might investors in practice arrive at different optimal risky portfolios than what the CAPM suggests? (Hint: Do all investors have the same constraints? The same universe of stocks they wish to invest in?)
20. If every investor held the market portfolio and no one conducted investment analysis, what might happen to market efficiency?
21. What does the CAPM predict about the relationship between beta and stocks returns? Historically, has this prediction come true?
22. What are "quintiles" of stocks as used when computing the performance of low and high beta stocks? What about when computing the performance of small and large stocks and value vs. growth stocks?
23. What assumptions does the CAPM made regarding the "homogeneity" of investors? Why are these assumptions necessary?
24. In what way do multifactor models improve the CAPM with regard to systematic risk measures?
25. If we compute a stock's expected return using either CAPM or Fama-French, why don't we include the stock's alpha (that we obtain by regression) in our calculations?
26. Why does the Fama-French 3-factor model specifically consider size and value?

27. If small stocks and value stocks have historically outperformed large and growth stocks, why don't we always just hold small and value stocks?
28. What is, for example, $E(r_{SMB})$ or $E(r_{HML})$? Where can you obtain these measures?
29. As we add more factors to the CAPM, what generally happens to alphas we calculate? What does this imply about securities' true "risk-adjusted return"?
30. As we add more factors to the CAPM, what happens to the R-squared? What does this imply about the multifactor model's "fit" relative to the CAPM?
31. Morningstar presents "CAPM alphas" on their website for different mutual funds. Which might an investor want to know the Fama-French alphas instead?
32. What anomalies do the UMD, CMA, and RMW attempt to account for in multifactor models? What motivates the addition of these models to the Fama-French multifactor model?
33. What do positive and negative "loadings" on different factors in the multifactor model imply about a stock or security's "exposure" to those sources of systematic risk?
34. If a stock has a large positive SMB beta, does it mean it is a small stock? If a stock has a large positive HML beta, does it mean it is a value stock?
35. CHALLENGE Explain when we'd see a negative slope for the SCL and a negative intercept for the SCL. Are negative slopes and negative intercepts possible in practice? Could we see both for a stock at the same time?
36. CHALLENGE A mutual fund manager achieves high alpha over the course of a year. As a result, the manager's fund experiences large **inflows** of money in the subsequent year. Thousands of new investors think she will continue to be an excellent manager and generate alpha in the future. How might this large inflow of capital to her fund make it challenging for her to generate alpha in subsequent periods?
37. CHALLENGE Value stocks have outperformed growth stocks over a long period of time. What are some arguments as to why they are riskier than growth stocks (which explains why they've performed well)?
38. CHALLENGE **Smart beta ETFs** use a rules-based approach to build portfolios that have exposure to selected factors (such as size, momentum, profitability, and so on). How might they achieve "exposure" to some of the factors we've talked about? If, for example, you have a smart beta ETF that seeks exposure to the value anomaly, how could you run a regression to determine if the ETF achieves its goal? How are smart beta ETFs a blend of active and passive investing?

ANALYTICAL QUESTIONS

1. Below is an output from Morningstar for the actively-managed Bridgeway Small Cap Value Fund. Use this output to answer the questions that follow.

Risk & Volatility Measures ⓘ

Trailing	Fund	Category	Index
Alpha	-0.60	-9.44	-8.76
Beta	1.19	1.31	1.26
R^2	58.18	76.35	77.23
Sharpe Ratio	0.61	0.39	0.40
Standard Deviation	29.42	28.17	26.99

USD | Fund as of Sep 30, 2021 | Category: Small Value as of Sep 30, 2021 | Index: Russell 2000 Value TR USD as of Sep 30, 2021 | Calculation Benchmark: S&P 500 TR USD

   a. What is this security's "reward per unit of risk"? Verify this number given the arithmetic average return of this fund's annual return over three years was 18.07%, and assume the risk-free rate is 0%.
   b. What is this security's expected return when the market excess return is 0%?
   c. What is the portion of this fund's return variation explained by the market?
   d. What is the portion of this fund's return variation explained by fund-specific factors?
   e. If the market rises 2%, how much do we expect this fund will rise?
   f. Did this fund outperform or underperform other funds in its category?
   g. If this fund's expected return is 18.07% per year, how often do we expect annual returns between 47.49% and -11.35%? Is a return of 0% or less feasible and/or reasonable?

2. You run a regression of an ETF's monthly excess return on the monthly market, size, and value factors to generate the following output. Use this output to answer the questions that follow. Visit *Python Code: Multifactor Model Regressions* and run the code if you'd like to view additional examples.

OLS Regression Results

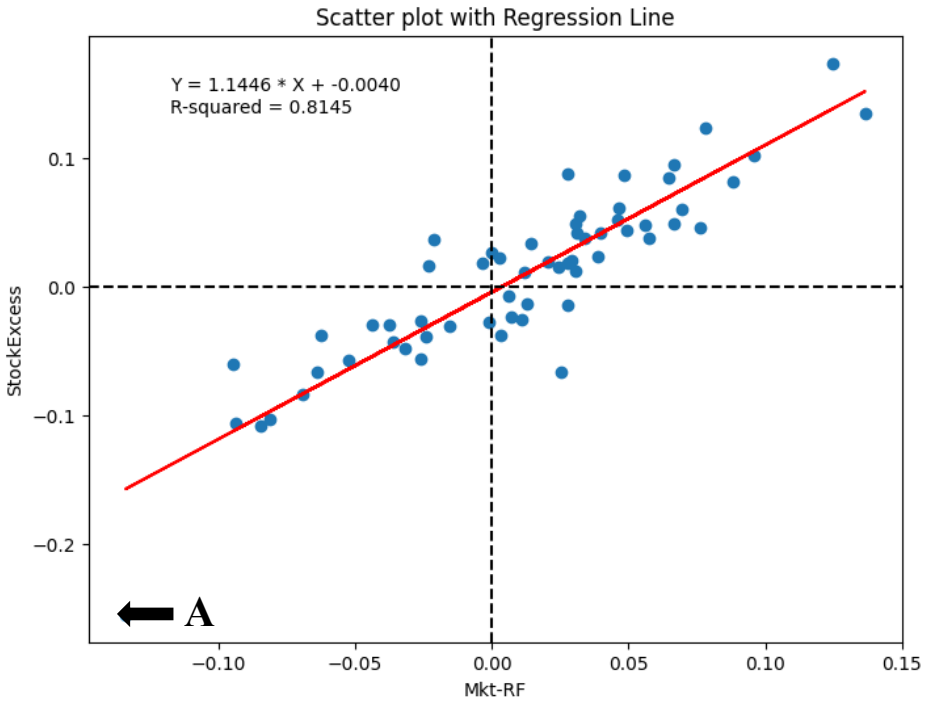
Dep. Variable:	StockExcess	R-squared:	0.977
Model:	OLS	Adj. R-squared:	0.976
Method:	Least Squares	F-statistic:	795.9
Date:	Mon, 18 Mar 2024	Prob (F-statistic):	7.26e-46
Time:	22:03:30	Log-Likelihood:	188.65
No. Observations:	60	AIC:	-369.3
Df Residuals:	56	BIC:	-360.9
Df Model:	3		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-0.0015	0.001	-1.054	0.297	-0.004	0.001
Mkt-RF	1.0259	0.027	37.511	0.000	0.971	1.081
SMB	0.4892	0.050	9.796	0.000	0.389	0.589
HML	0.4042	0.032	12.619	0.000	0.340	0.468

Omnibus:	0.517	Durbin-Watson:	2.354
Prob(Omnibus):	0.772	Jarque-Bera (JB):	0.647
Skew:	0.083	Prob(JB):	0.724
Kurtosis:	2.519	Cond. No.	38.0

a. Was this ETF undervalued or overvalued by the Fama-French model given its risk adjusted return?
b. Does this ETF appear to hold small or large stocks in its portfolio?
c. Does this ETF appear to hold value or growth stocks in its portfolio?
d. Does this security appear to be more or less volatile than the market? Or is its volatility about the same as the market?

3. You plot a security's monthly excess returns and the market's excess returns at *Python Code: Multifactor Model Regressions* and generate the following output. Use this output to answer the questions that follow.



a. Explain what is happening at point A in the diagram.
b. What is the name of the red line passing through these points?
c. Explain what Y and X in the equation at the top left of the diagram represent.
d. If the market returns 5% in a month when T-bills yield 2%, what do we expect this security will return?
e. What is this security's beta?
f. What is this security's alpha?
g. Given this security's alpha, do CAPM assumptions imply investors will rush to buy or sell this security in order for it to be appropriately priced?
h. What percentage of this security's return variation is explained by the market?
i. What percentage of this security's return variation is explained by security-specific factors?

CFA QUESTIONS

Answers are in the *Notes & References* section below.[5]

1. The line depicting the total risk and expected return of portfolio combination of a risk-free asset and any risky asset is the:
    a. Security market line
    b. Capital allocation line
    c. Security characteristic line
2. The portfolio of a risk-free asset and a risky asset has a better risk-return tradeoff than investing in only one asset type because the correlation between the risk-free asset and the risky asset is equal to
    a. -1.0
    b. 0.0
    c. 1.0
3. Relative to portfolios on the CML, any portfolio that plots above the CML is considered:
    a. Inferior
    b. Inefficient
    c. Unachievable

Use the table below to answer questions 4 and 5.

Security	Expected Annual Return (%)	Expected Standard Deviation (%)	Correlation between Security and the Market
Security 1	11	25	0.6
Security 2	11	20	0.7
Security 3	14	20	0.8
Market	10	15	1.0

4. Which security has the *highest* beta measure?
    a. Security 1
    b. Security 2
    c. Security 3
5. Which security has the *least* amount of market risk?
    a. Security 1
    b. Security 2
    c. Security 3
6. The slope of a security characteristic line is an asset's
    a. Beta
    b. Excess return
    c. Risk premium

7. Analysts who have estimated returns of an asset to be greater than the expected returns generated by the CAPM should consider the asset to be:
    a. Overvalued
    b. Undervalued
    c. Properly valued
8. With respect to the CAPM, the primary determinant of expected return of an individual asset is the
    a. Asset's beta
    b. Market risk premium
    c. Asset's standard deviation

Use the table below to answer questions 9 through 12.

Security	Expected Standard Deviation (%)	Beta
Security 1	25	1.50
Security 2	15	1.40
Security 3	20	1.60

9. With respect to the capital asset pricing model, if the expected market risk premium is 6% and the risk-free rate is 3%, the expected return for Security 1 is closest to:
    a. 9.0%
    b. 12.0%
    c. 13.5%
10. With respect to the capital asset pricing model, if expected return for Security 2 is equal to 11.4% and the risk-free rate is 3%, the expected return for the market is closest to:
    a. 8.4%
    b. 9.0%
    c. 10.3%
11. With respect to the capital asset pricing model, if the expected market risk premium is 6% the security with the highest expected return is:
    a. Security 1
    b. Security 2
    c. Security 3
12. With respect to the capital asset pricing model, a decline in the expected market return will have the greatest impact on the expected return of:
    a. Security 1
    b. Security 2
    c. Security 3

NOTES & REFERENCES

[1] If $n = 3000$, then $(n^2 - n)/2 = 4{,}498{,}500$ covariances, 3000 expected returns, and 3000 standard deviations.
[2] 72% of valuation professionals always or almost always use the CAPM: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2784850
[3] Ken French's website: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[4] WRDS: https://wrds-www.wharton.upenn.edu/
[5] CFA Question answers: 1)B, 2)B, 3)C, 4)C, 5)B, 6)A, 7)B, 8)A, 9)B, 10)B, 11)C, 12)C