University of Richmond
Robins School of Business™

§8. Capital Allocation

FIN 366: Investments

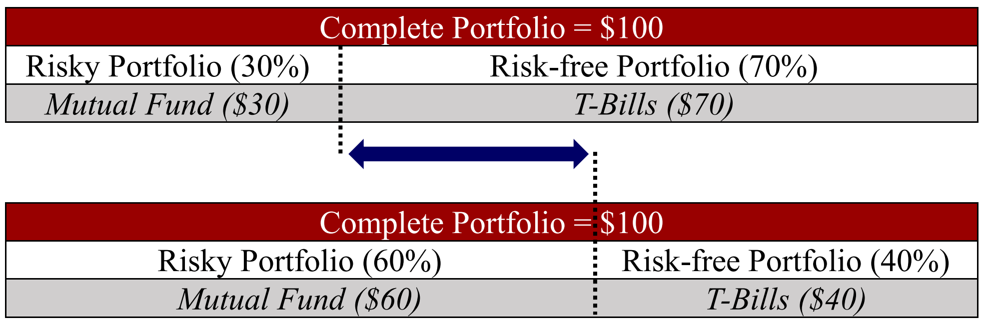
TABLE OF CONTENTS

Capital Allocation to Risky Assets

Capital Allocation

Given an understanding of risk and reward, we can begin to consider allocation of capital across a risky portfolio (say, a mutual fund) and a risk-free portfolio (for example, T-Bills). Investors can adjust the risk and return profile of a **complete portfolio** by adjusting the allocation of their capital to the risky portfolio and to the risk-free portfolio.

Figure 1: The Complete Portfolio



Capital Allocation to Risky Assets is making the choice of how much to invest in the risky portfolio relative to the risk-free portfolio. An investor's own **risk tolerance** informs this choice.

The expected return of the complete portfolio $\mathbf{E(r_c)}$ is the weighted average of the expected return of the risky portfolio $\mathbf{E(r_p)}$ and the return of the risk-free asset $\mathbf{r_f}$, given you invest **y** percent of your money in the risky portfolio:

$$E(r_c) = yE(r_p) + (1 - y)r_f$$

The standard deviation, our risk measure, of the complete portfolio $\boldsymbol{\sigma_C}$ is the standard deviation of the risky portfolio $\boldsymbol{\sigma_p}$ multiplied by its weight **y**:

$$\sigma_c = y\sigma_p$$

Why is the formula $\boldsymbol{\sigma_c = y\sigma_p}$ and not $\boldsymbol{\sigma_c = y\sigma_p + (1-y)\sigma_{r_f}}$ for the overall risk of the complete portfolio?

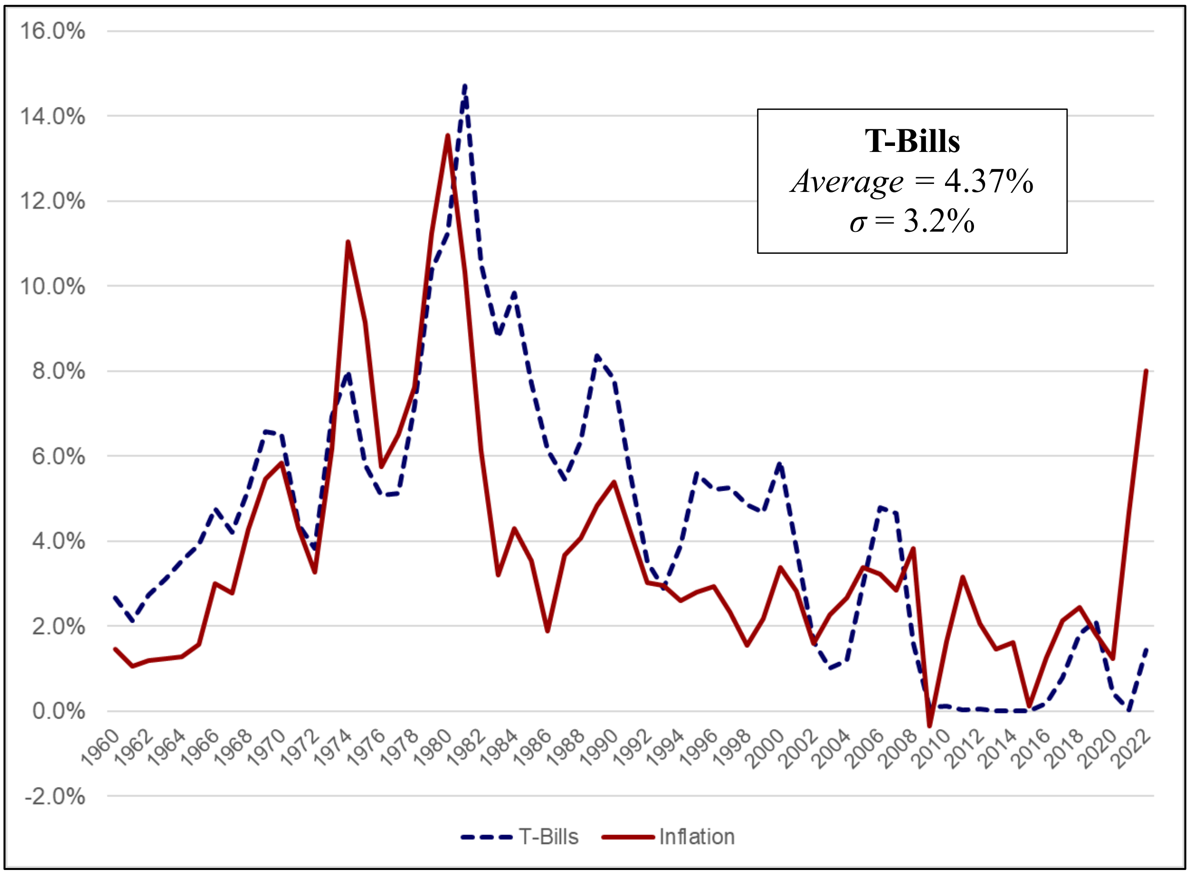
____

____

The Risk-Free Asset: T-Bills

The return on T-bills technically *has* a standard deviation. However, we treat this as *"changes to the risk-free rate over time"* rather than risk.

Figure 2: Inflation and Annual Return on T-bills



View the Excel File and data sources that generated this figure at *Inflation and the Risk Free Rate* at josephfarizo.com/fin366.html.

PRACTICE: What is the (a) expected return, (b) standard deviation, (c) risk premium, and (d) Sharpe ratio for a complete portfolio given the yield on 90-day T-Bills is 3.7%, the expected return of a risky fund is 11.93%, its standard deviation is 20%, and you choose to hold 75% of your investing dollars in the risky portfolio?

SOLUTION: The expected return (a) of the complete portfolio is the weighted average of the returns to the risky portfolio and risk-free asset:

$$E(r_c) = yE(r_p) + (1 - y)r_f$$

The standard deviation of the complete portfolio (b) is:

$$\sigma_c = y\sigma_p$$

Recalling the definition of **risk premium** as the expected return in excess of the return on risk-free securities, (c) must be:

$$E(r_c) - r_f =$$

And we use this risk premium for the Sharpe Ratio computation (d):

$$S = \frac{E(r_C) - r_f}{\sigma_C} = \frac{Risk\ Premium}{\sigma_c} =$$

Note that we now subscript the expected return and standard deviation with *C* since we are computing this ratio for the complete portfolio.

INTERPRETATION: The complete portfolio's risk and return profile is affected by the weights we put in the risky asset. Increasing weight to the risky portfolio increases the return, but it also increases the risk. *But what happens to the Sharpe ratio as we increase the weight 'y' in the risky asset?*

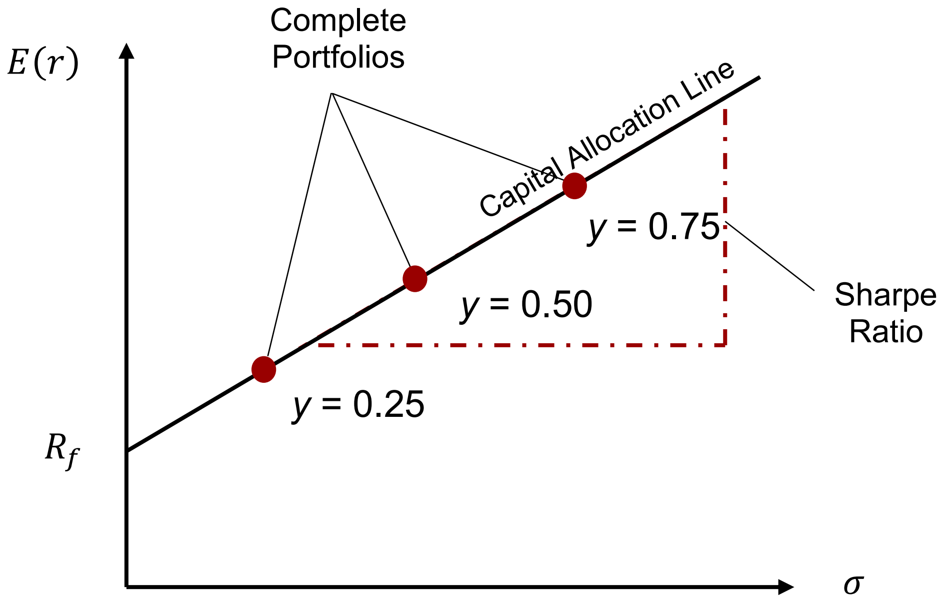
Additional practice is available in the Excel file *Capital Allocation* posted to josephfarizo.com/fin366.html.

THE CAPITAL ALLOCATION LINE

The complete portfolio can have varying risk-return profiles by changing the weight *y* in the risky asset. We can plot this on a coordinate plane, with the expected return on the vertical axis, and the risk on the horizontal axis.

Each point tells us the return and risk of a portfolio that consists of the risky asset and the risk-free asset. As we adjust the weight *y*, the return and risk profile changes linearly. All complete portfolios will fall on one line, with the slope of the line the Sharpe ratio.[1]

Figure 3: The Capital Allocation Line



The various complete portfolios we plot comprise the **investment opportunity set** achievable by varying *y*. The line drawn through these portfolios is known as the **Capital Allocation Line (CAL)**. The CAL's intercept with the vertical axis is the risk-free rate of return (where we assume a standard deviation of 0.)

> ★ All portfolios on the CAL have the same Sharpe ratio. Therefore, no portfolio on the CAL dominates another portfolio on the same CAL. Varying *y* proportionally changes risk and return.

Example: Generate additional CALs using the Excel file *Capital Allocation Line* available at josephfarizo.com/fin366.html. Below are axes to draw on and label.

Interpretation: Different CALs are possible if you change the risky asset itself. Moving *along* a CAL is achieved by changing the weights between the risk-free asset and a selected risky asset.

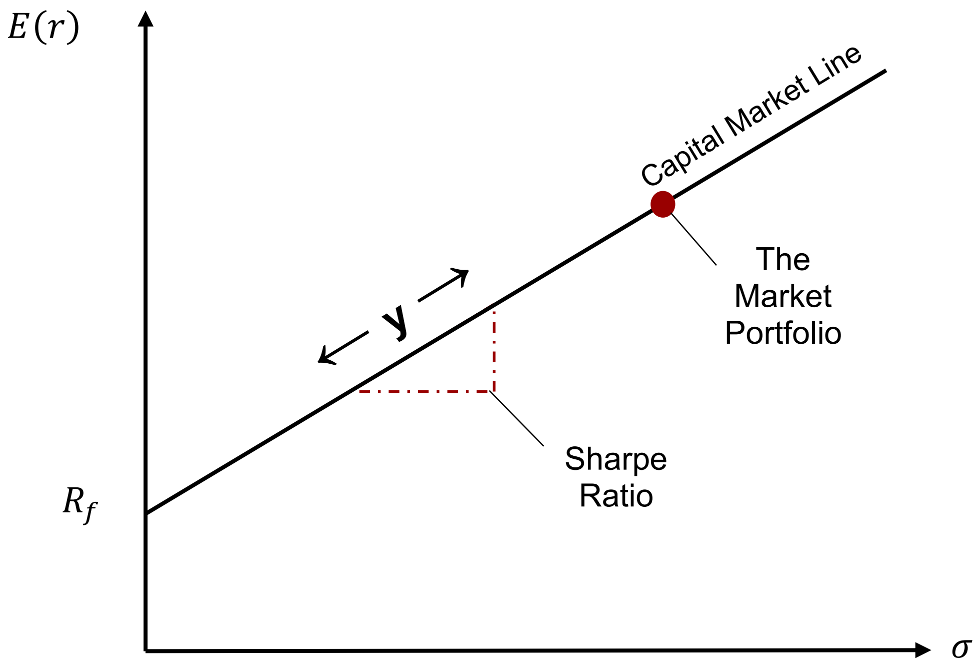
The Capital Market Line & The Passive Strategy

The CAL consists of some weight y in a risky portfolio and $1-y$ in the risk-free asset. This risky portfolio can be an actively managed or passively managed mutual fund. It might be a portfolio of stocks, bonds, or other assets that an investor constructs themselves.

The Capital Market Line

The **Capital Market Line** (**CML**) is a special case of the CAL where we take the *entire market portfolio* to be the risky portfolio held in combination with the risk-free asset. The market portfolio is a theoretical value-weighted portfolio of every asset, whether publicly traded or not: stocks, bonds, currencies, real estate, fine art, automobiles, collectibles, etc.

Figure 4: The Capital Market Line



The market portfolio is a theoretical portfolio that cannot be held. If investors want to get close to the CML, what can they do?

____

____

THE PASSIVE STRATEGY

Pursuing a passive strategy entails allocating capital between the risk-free asset and the market portfolio rather than individually selecting how to construct the risky portfolio. As we've seen above, any choice of y is as good as any other and should be chosen based on the investor's risk tolerance. The passive strategy is a reasonable strategy for two main reasons:

1. **Costs of Active Investing**

   An investor must spend time and money to pursue an active strategy, as well as pay fees to investment professionals. The passive strategy only entails choosing your weight y given some risky portfolio.

2. **The Free-Rider Benefit**

   Given the large number of active buyers and sellers of securities in markets, we expect that knowledgeable investors quickly bid up the prices of undervalued stocks and force down the prices of overvalued stocks (through selling), leaving most assets with a fair price at any given time by market efficiency.

> A portfolio of assets that are fairly priced by market efficiency is itself priced fairly.

CRITICAL THINKING QUESTIONS

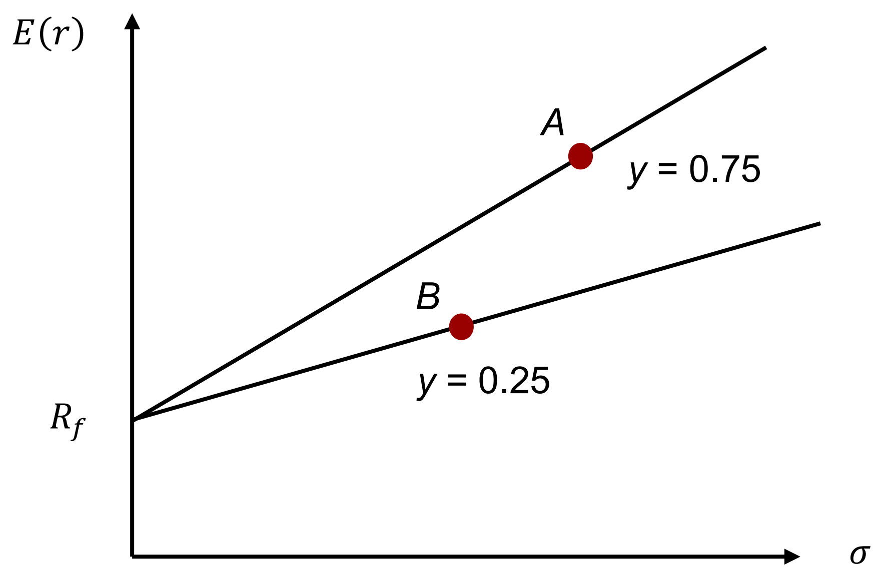
1. What is the "capital allocation" decision?
2. Explain how T-bills are risk-free if they have a historical standard deviation.
3. A friend tells you "I have 90% of my money in the QQQ index fund, and the rest in cash. You only have 60% of your money in the QQQ index fund and the rest in cash. Clearly my expected return is higher, and my portfolio is superior to yours." Critique this claim.
4. What determines how much an investor invests in the risky and risk-free portfolios?
5. How many different portfolios are there in the **investment opportunity set** that the CAL graphically depicts?
6. How do we obtain the expected return and standard deviation of the risky portfolio that is needed to compute the expected return and standard deviation of the complete portfolio?
7. What type of assets comprise a portfolio on the CAL?
8. What might we use as the "risky asset"?
9. Is there a "best" portfolio to hold on the CAL?
10. Can you draw and label the CAL?
11. Can you draw and label the CML?
12. What is the slope of the CAL? What is the slope of the CML?
13. Can you derive the slope of the CAL given the coordinates (x_2, y_2) for the complete portfolio and the coordinates (x_1,y_1) for the risk-free rate?
14. What can cause the CAL's slope to increase? (Hint: use the CAL Excel File)
15. What can cause the intercept of the CAL to shift upward? (Hint: use the CAL Excel File)
16. What can "elongate" or "stretch" the CAL? (Hint: use the CAL Excel File)
17. What is the y-intercept of the CAL?
18. What could we use to proxy for the market portfolio on the CML, and why do we need to "proxy" for the market portfolio?
19. If we only know the expected return of the risky portfolio, the standard deviation of the risky portfolio, and the risk-free rate of return, can we find the Sharpe ratio of the *complete* portfolio *without* knowing the weight we put in the risk-free asset?
20. How can two investors have two different CALs?
21. Why do all investors have the same CML?
22. A friend tells you "I have 90% of my money in the QQQ index fund with an expected return of 22%, and the rest in cash. You have 90% of your money in the SPX index fund with an expected return of 14% and the rest in cash. Clearly my portfolio is superior to yours." Critique this claim.
23. CHALLENGE **Risk Aversion** can vary from person to person. If, for example, you have a higher degree of risk aversion than your friend does, what does that imply about the level of tolerance you have for a portfolio's risk relative to your friend (assuming the portfolio's return is held constant)?
24. CHALLENGE An investor holds some of their money in the Fidelity Small Cap Growth Fund and some of their money in T-bills. Another investor holds some of their money in the Vanguard Large Cap Value Fund and some in T-bills. Do they have the same Sharpe ratio? *Can* they have the same Sharpe ratio? Do these investors lie on the same CAL? Can one of

these investors have a better "complete" portfolio than the other just by changing the weights they invest in their risky portfolio?

25. CHALLENGE If our Sharpe ratio doesn't change based on how much we invest in the risky portfolio versus how much we invest in the risk-free asset, why does picking different securities, mutual funds, and investments matter at all? (Hint: it does matter!)
26. CHALLENGE How many different CALs are possible across different investors? How many different CALs can you as an individual have at a given point in time? How many different CALs can you have across time?

ANALYTICAL QUESTIONS

1. Use the diagram of the CALs below to answer the questions that follow. The portfolio labeled *A* consists of an allocation between the risk-free asset and a stock portfolio. The portfolio labeled *B* consists of an allocation between the risk-free asset and a bond portfolio. Warren holds the stock portfolio with y = 0.75 and Charlie holds the bond portfolio with y = 0.25.



a. What can Warren do if he wants to lower his risk below that of Charlie?
b. Is it possible for Charlie to increase his return beyond that of Warren?
c. Can we conclude that one of these portfolios is better than the other?
d. If Warren lowers his y to 0.10 and Charlie raises his y to 0.9, who will have the higher Sharpe ratio?

CFA QUESTIONS

Answers are in the *Notes & References* section below.[2]

1. Which of the following is *least* accurate?
   a. Increasing capital allocation to the risky asset increases the Sharpe ratio
   b. Increasing capital allocation to the risky asset increases expected returns
   c. Decreasing capital allocation to the risk-free asset increases expected returns

2. The slope of the capital allocation line can be increased
   a. If the investor chooses a different risky asset
   b. If the investor allocates more capital to the risky asset
   c. If the investor moves along the capital allocation line

NOTES & REFERENCES

[1] A helpful way of understanding why the Sharpe Ratio is the slope of the CAL is to recall the slope of a line on a standard Cartesian coordinate plane:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

We define our y and x axes as the return and standard deviation respectively. Given we choose point (x_2, y_2) to be the complete portfolio and point (x_1,y_1) to be the risk-free asset, we can rewrite our slope as:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{E(R_c) - r_f}{\sigma_c - \sigma_{r_f}}$$

By the assumption that the risk-free asset *is* risk free: $\sigma_{r_f} = 0$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{E(R_c) - r_f}{\sigma_c - \sigma_{r_f}} = \frac{E(R_c) - r_f}{\sigma_c} = S$$

[2] CFA Question answers: 1) A, 2) A