

# **§12. EQUITY VALUATION: INTRINSIC VALUES AND DIVIDEND DISCOUNT MODELS**

FIN 366: INVESTMENTS  
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## VALUING EQUITY APPROACHES

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Market efficiency implies that shares generally trade at appropriate prices based on all available information. We now consider different approaches, specifically *fundamental analysis* tools, that market participants use to arrive at “appropriate” values for shares of common stock.

Recall that **fundamental analysis** seeks to identify stocks that differ from their theoretical true or *intrinsic* value- buy the undervalued and sell/short the overvalued.

## INTRINSIC VALUE VS. MARKET PRICE

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### EXPECTED VS. REQUIRED RETURNS

Assume a firm is a *going concern*, or a business that will continue to meet its obligations and operate indefinitely. Recall the return to a stock investment in such a firm includes the **capital gains** and **dividend yields**. An **expected holding period return** would be represented as:

$$\text{Expected HPR} = E(r) = \frac{E(P_1) - P_0 + E(Div_1)}{P_0} = \frac{E(P_1) - P_0}{P_0} + \frac{E(Div_1)}{P_0}$$

where  $P$  is the stock’s price and  $Div$  is the per-share dividend. For now, we take the  $E(P_1)$  as given, highlighting how important model inputs are for our calculations.

An investor can compare the **expected HPR** to some **required rate of return** that a stock should yield given its level of risk.

- *Expected = Required*: stock fairly priced
- *Expected > Required*: stock undervalued → buy
- *Expected < Required*: stock overvalued → sell/short

To determine the required rate of return, we consider again the **Capital Asset Pricing Model**

$$k = r_f + \beta[E(R_M) - r_f]$$

which gives us the return  $k$  we *should require* of an asset given its level of risk  $\beta$ .

## INTRINSIC VALUE

Given the same inputs (expected price, expected dividend, and  $k$ ) required to compute the expected and required returns, we can compute an estimate of a stock's **intrinsic value**,  $V_0$ :

$$V_0 = \frac{E(P_1) + E(Div_1)}{1 + k}$$

That is, the value of the stock today is the *present value* of all cash inflows including dividends and ultimate proceeds of the sale of stock during the holding period. The discount rate is one plus the required rate of return by the CAPM.

- $V_0 = P_0$ : stock fairly priced
- $V_0 > P_0$ : stock undervalued → buy
- $V_0 < P_0$ : stock overvalued → sell/short



**PRACTICE:** A firm's share price is currently \$105, and you expect it to be trading at \$108 at the end of your holding period, at which time you expect to receive a \$3 dividend. This stock's beta by the index model is 0.9, and you expect the market return to be 13%. Given T-bills yield 3%, are these shares over-, under-, or appropriately valued?

The Excel file [Expected vs. Required Return](http://josephfarizo.com/fin366.html) available at [josephfarizo.com/fin366.html](http://josephfarizo.com/fin366.html) provides a walkthrough of this example and other practice problems.

**SOLUTION:** After identifying the inputs, we obtain the expected HPR:

$$\begin{aligned} \text{Expected HPR} = E(r) &= \frac{E(P_1) - P_0 + E(Div_1)}{P_0} = \frac{\quad - \quad + \quad}{\quad} \\ &= 5.7\% \end{aligned}$$

Then, we compute the required rate of return

$$k = r_f + \beta[E(R_M) - r_f] = \quad + \quad [ \quad - \quad ] = 12.0\%$$

And comparing the two, we see that the expected return is \_\_\_\_\_ than the required return, implying these shares are \_\_\_\_\_ valued. We should therefore \_\_\_\_\_ these shares.

Similarly, if we compute an estimate of the stock's intrinsic value:

$$V_0 = \frac{E(P_1) + E(Div_1)}{1 + k} = \frac{\quad + \quad}{1 + \quad} = \$99.11$$

Which is \_\_\_\_\_ than the current market price of \$105, implying this share is \_\_\_\_\_.

**INTERPRETATION:** Given the same inputs, we should reach the same “under” or “over” valued conclusion from the “expected vs. required returns” and “intrinsic vs. market value” methods.

## **DIVIDEND DISCOUNT MODEL**

The **Dividend Discount Model (DDM)** equates the intrinsic value of a stock to the present value of all future dividends paid to the stockholder on that stock. We will consider multiple cases:

- i. Dividends over a discrete period (one → many)
- ii. Constant dividend growth
- iii. Multistage growth (begins as i., then becomes ii.)

## DDM FOR ONE PERIOD

We have established the intrinsic value for one holding period is:

$$V_0 = \frac{D_1 + P_1}{1 + k}$$

For simplicity, we omit the  $E$  for expectations in our notation. That is, the intrinsic value today is the present value of the end-of-period dividend and share price.

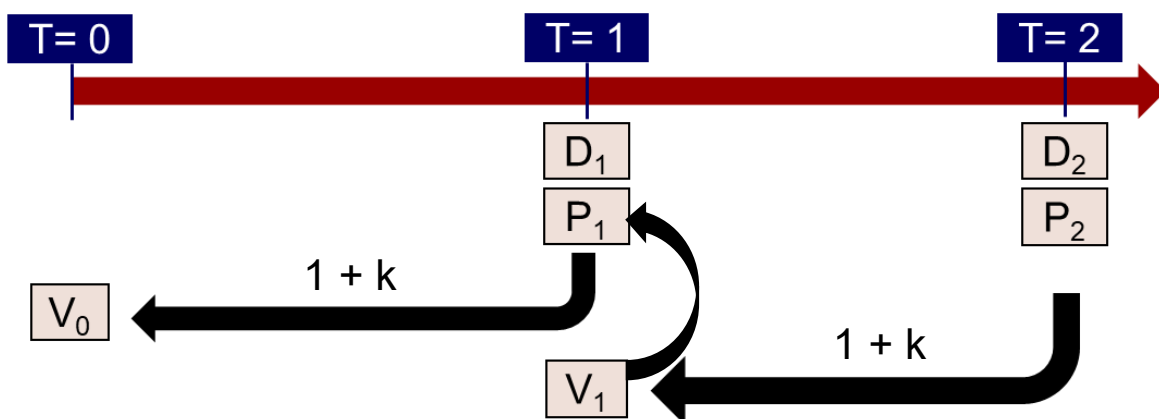
## DDM FOR TWO PERIODS

By the same logic (and by “stepping up the subscripts”), the intrinsic value at time  $T = 1$  would be

$$V_1 = \frac{D_2 + P_2}{1 + k}$$

Our timeline becomes (under the assumption that the stock will be trading at its intrinsic value next year, assume  $P_1 = V_1$ ):

*Figure 1: DDM Timeline*



We substitute  $V_1$  for  $P_1$  in the equation for  $V_0$ , and  $V_1 = \frac{D_2 + P_2}{1+k}$ , which results in:

$$V_0 = \frac{D_1}{1+k} + \frac{D_2 + P_2}{(1+k)^2}$$

### DDM FOR MANY PERIODS

We generalize the two-period formula for many periods by repeating the same process that we used to generalize from the one period DDM to the two period DDM. For  $H$  periods:

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \dots + \frac{D_H + P_H}{(1+k)^H}$$

Doing this *indefinitely* yields

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \dots$$



*Where is the Price  $P_H$ ?* In the indefinite period model, the price  $P_H$  does not appear. This does not imply that capital gains are irrelevant. The price  $P_H$  itself is just the present value of all future dividends and is “embedded” in the series of future dividend payments. This assumes that all stocks will at some point pay a dividend, even if only a liquidating dividend.



**PRACTICE:** A firm is expected to pay dividends of \$4, \$6, and \$8 over the next 3 years. If you expect to sell a share at \$100 after collecting your \$8 dividend in the third year, what is the value of the share today? By the CAPM, the required rate of return on this stock is 10%.

**SOLUTION:** Using the formula

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \dots + \frac{D_H + P_H}{(1+k)^H}$$

where  $H = 3$ , we have

$$V_0 = \frac{D_0}{1+k} + \frac{D_0(1+g)}{(1+k)^2} + \frac{D_0(1+g)^2}{(1+k)^3} + \dots = \$89.73$$



If the firm in the previous example trades at \$93, what should the investor do? Why?

### DDM FOR CONSTANT GROWTH DIVIDENDS

Forecasting dividends payments for several periods and/or indefinitely is challenging. Alternatively, we might assume a dividend *just paid*  $D_0$  grows at a constant rate  $g$ . This model is referred to as the **Gordon Growth Model**.

$$V_0 = \frac{D_0(1+g)}{(1+k)} + \frac{D_0(1+g)^2}{(1+k)^2} + \dots = \frac{D_1}{k-g}$$

or<sup>1</sup>

$$V_0 = \frac{D_0(1+g)}{k-g} = \frac{D_1}{k-g}$$



Notice that we start with the dividend  $D_0$  that was just paid.  $D_1$  is the *next* period's dividend, equivalent to the "just paid" dividend grown for one period at the growth rate  $g$  in the numerator.

This model also requires  $k > g$ . That is, the growth rate in dividends must be less than the required rate of returns on the stock. If  $k < g$ , it implies that dividends are growing faster than the rate of return required for this stock given its risk. This would be unsustainable in the long term.



### The Constant Growth Rate $g$

The growth rate  $g$  may use the historical growth rate of dividends that the firm has paid over the past several years, analyst forecasts, or the formula:

$$g = ROE \times b$$

**ROE** is the **return on equity**, and  $b$  is the **plowback** or **retention ratio**, or the proportion of net income that the firm retains and reinvests rather than distributing to shareholders:

$$g = \frac{\text{Net Income}}{\text{Equity}} \times (1 - \text{Dividend Payout Ratio})$$

and the **dividend payout ratio** is the total dividends the firm paid divided by net income.



**PRACTICE:** A firm has net income of \$100,000 and book value of equity on the balance sheet of \$1 million. It paid 70% of its net income out in dividends, amounting to \$4 per share. If the required rate of return = 13%, what is an estimate of the intrinsic value of the firm's shares?

**SOLUTION:** By the formula

$$V_0 = \frac{D_0(1 + g)}{k - g} = \frac{D_1}{k - g}$$

and

$$g = ROE \times b = \quad \times \quad = 0.03 = 3\%$$

we have

$$V_0 = \frac{(1 + \quad)}{\quad} = \frac{4.12}{\quad} = \$41.20$$

### MULTISTAGE GROWTH DDM

In the multistage growth model, we forecast dividends for a fixed period of **supernormal growth**, then assume dividend growth will level off to some **steady growth** rate  $g$  in the future. This combines the two previous methods of discrete dividend payments and the constant growth model.

Beginning with the formula for discrete dividend payments:

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \dots + \frac{D_H + P_H}{(1+k)^H}$$

We can substitute the constant growth formula in for  $P_H$

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \dots + \frac{D_H + \frac{D_{H+1}}{k-g}}{(1+k)^H}$$



**PRACTICE:** A firm just paid a dividend of \$1 that is expected to grow 20% year-over-year for the next 2 years, then 2% thereafter. Given a required return of 8.5%, what is the estimate of the firm's intrinsic value?

**SOLUTION:** By the formula

$$V_0 = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \dots + \frac{D_H + \frac{D_{H+1}}{k-g}}{(1+k)^H}$$

we have the following for this two-period problem:

$$V_0 = \frac{D_1}{1+k} + \frac{D_2 + \frac{D_3}{k-g}}{(1+k)^2}$$

and

$$D_0 =$$

$$D_2 =$$

$$D_1 =$$

$$D_3 =$$

The solution is:

$$V_0 = \frac{1 \times (1 + )}{1 + } + \frac{1 \times (1 + )^2 + \frac{1 \times }{ - } \times }{(1 + )^2} = \$21.52$$



The Excel file [DDM](http://josephfarizo.com/fin366.html) available at [josephfarizo.com/fin366.html](http://josephfarizo.com/fin366.html) provides additional practice problems for the dividend discount model.



In practice, the **steady growth rate**  $g$  should not exceed 2-3%, the approximate long-term growth rate of the economy. Having a steady growth rate  $g$  that exceeds the growth rate of the economy would imply that the size of the firm would eventually exceed the size of the economy.

## APPLYING THE DIVIDEND DISCOUNT MODEL

Obviously, a firm must pay dividends (or be expected to pay dividends) to be valued by the dividend discount model. The firms that are the most viable candidates for valuation by the DDM include:

1. Firms with high and consistent dividend payouts (dividend yields exceeding 3-5%).
2. Firms with steady cash flows.
3. Mature firms with steady, albeit “low to moderate,” growth.
4. Firms in highly regulated industries, including utilities.
5. **Real estate investment trusts (REITs)**. These are exchange-tradable real estate portfolios required by law to pay out at least 90% of its income as dividends.<sup>2</sup>
6. Financial services firms, such as banks and insurance companies. Given their high leverage and regulatory requirements, other discounted cash flow techniques we will learn about are not appropriate for these firms. The DDM is a suitable approach.
7. **Master limited partnerships (MLPs)**. These are exchange-tradable organizations that usually focus on the exploration and processing of natural resources such as oil and gas. By law, these organizations must pay out at least 90% of their income as dividends.<sup>3</sup>

In practice, the multistage growth model with about 3-5 years of supernormal growth followed by steady growth, tends to be the most commonly employed method of the dividend discount model.



All valuation methods are sensitive to their inputs. Different analysts can and do frequently arrive at different estimates of intrinsic value, which in part explains what *makes the market* in equity securities.

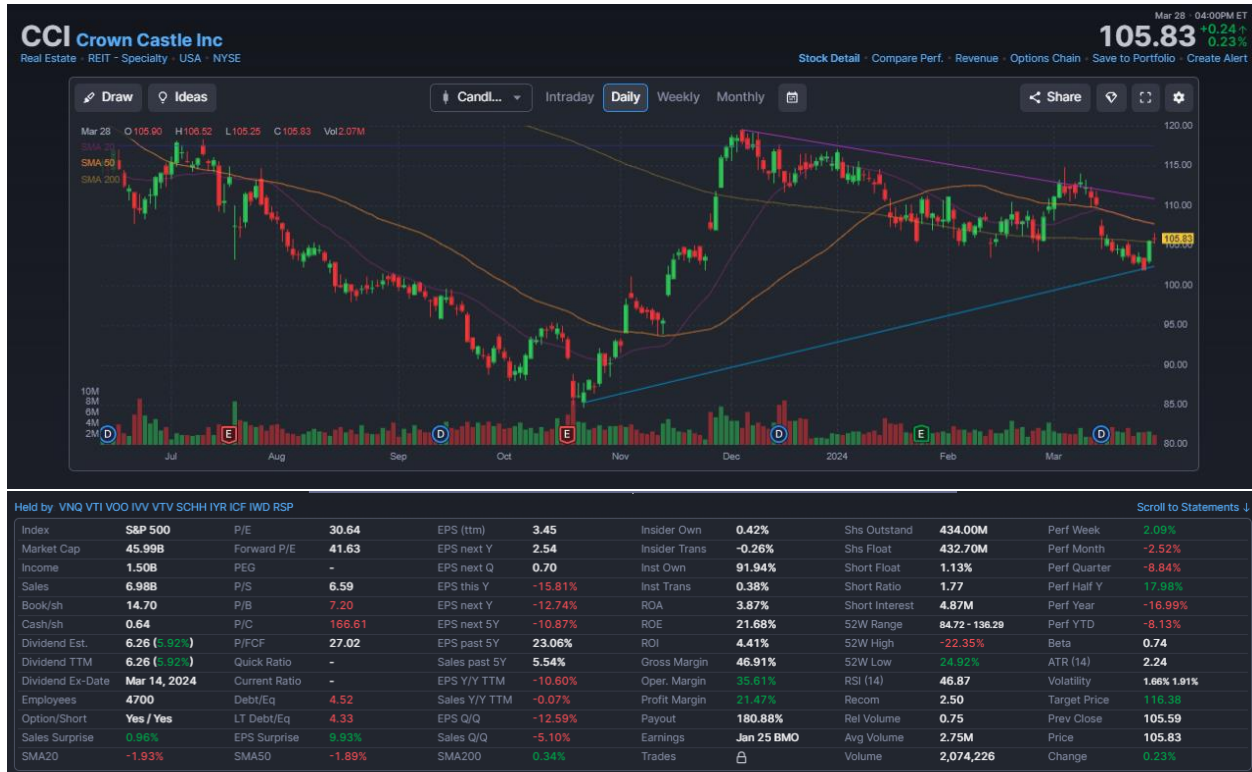
## CRITICAL THINKING QUESTIONS

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1. How should the relationship of the intrinsic value we estimate and the stock's current price inform our investing decisions?
2. Why would it make sense to buy a share that is undervalued or short a share that is overvalued? What do we think the market will "realize" or "figure out" about undervalued and overvalued stocks in the future?
3. How do we compute the required rate of return on a security?
4. How do we compute the growth rate  $g$  for a firm?
5. What should you do if the required rate of return is greater than the return you expect on a security? Why?
6. We can determine the value of a share by taking the present value of an indefinite series of dividends. If we do this, why don't we need to know what the stock price is in the future?
7. A younger firm has experienced a high rate of dividend growth recently, though this is expected to fall to a steadier rate in the future. Do you think a (1) dividends over a discrete period model, (2) constant growth model, or (3) multistage growth model of the DDM would be best to value these shares?
8. A mature firm has a history of steady dividends over the past several years. Do you think the (1) dividends over a discrete period model, (2) constant growth model, or (3) multistage growth model of the DDM would be best to value these shares?
9. In practice, which is most commonly employed? (1) Dividends over a discrete period, (2) constant growth, or (3) multistage growth model?
10. In the two period DDM model, what assumption regarding  $P_I$  and  $V_I$  do we make?
11. In the multistage growth model, we may substitute the constant growth formula as the price of the stock  $P_H$  in the future. What assumption regarding  $P_H$  and  $V_H$  do we make for this model to be possible?
12. The constant growth DDM requires that  $k$  be greater than  $g$ . Why? What would happen if  $k$  were less than  $g$ ?
13. Look at the formulas for the DDM. Will the intrinsic value of a share increase or decrease in each of the following situations (assume everything else is held constant):
  - a.  $k$  increases
  - b. Beta increases
  - c. The market risk premium increases
  - d. You forecast a higher  $P_H$
  - e. You forecast a higher  $g$
  - f.  $ROE$  increases
  - g.  $b$  increases (Hint: explain why this one may be indeterminate)
14. **CHALLENGE** Explain the characteristics of a firm that would make it a suitable candidate for valuation by the dividend discount model. Which characteristics of firms would make it an *unsuitable* candidate for valuation by the dividend discount model?
15. **CHALLENGE** Explain why different conclusions on estimates of intrinsic values is necessary to make a market for equity securities.

## ANALYTICAL QUESTIONS

Show that an estimate of the intrinsic value of the company below is approximately \$97.25 by the Gordon Growth Model. Note that the dividend reported is the *estimated* next period dividend. The S&P 500 has yielded about 9.75% each year over the last 20 years, and the current yield on 90-day T-bills is about 5.345%. Assume this firm, as a REIT, pays out the mandatory 90% of its income in dividends. Is this company over or undervalued by your estimates?



## CFA QUESTIONS

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Answers are in the *Notes & References* section below.<sup>4</sup>

1. An investor expects a share to pay dividends of \$3.00 and \$3.15 at the end of Years 1 and 2, respectively. At the end of the second year, the investor expects the shares to trade at \$40.00. The required rate of return on the shares is 8 percent. If the investor's forecasts are accurate and the market price of the shares is currently \$30, the most likely conclusion is that the shares are:
  - a. overvalued
  - b. undervalued
  - c. fairly valued
2. The current dividend,  $D_0$ , is \$5.00. Growth is expected to be 10 percent a year for three years and then 5 percent thereafter. The required rate of return is 15 percent. Estimate the intrinsic value.
  - a. \$59.68
  - b. \$55.40
  - c. \$50.01
3. An investor expects to purchase shares of common stock today and sell them after two years. The investor has estimated dividends for the next two years,  $D_1$  and  $D_2$ , and the selling price of the stock two years from now,  $P_2$ . According to the dividend discount model, the intrinsic value of the stock today is the present value of:
  - a. Next year's dividend,  $D_1$
  - b. Future expected dividends,  $D_1$  and  $D_2$
  - c. Future expected dividends and price,  $D_1$ ,  $D_2$ , and  $P_2$ .
4. The Beasley Corporation has just paid a dividend of \$1.75 per share. If the required rate of return is 12.3 percent per year and dividends are expected to grow indefinitely at a constant rate of 9.2 percent per year, the intrinsic value of Beasley Corporation stock is closest to:
  - a. \$15.54
  - b. \$56.45
  - c. \$61.65
5. The Gordon growth model can be used to value dividend-paying companies that are:
  - a. Expected to grow very fast
  - b. In a mature phase of growth
  - c. Very sensitive to the business cycle
6. An analyst is attempting to value shares of the Dominion Company. The company has just paid a dividend of \$0.58 per share. Dividends are expected to grow by 20 percent next year and 15 percent the year after that. From the third year onward, dividends are expected to grow at 5.6 percent per year indefinitely. If the required rate of return is 8.3 percent, the intrinsic value of the stock is closest to:
  - a. \$26.00
  - b. \$27.00
  - c. \$28.00

## NOTES & REFERENCES

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<sup>1</sup> Proof: Given

$$V_0 = \frac{D_1}{1+k} + \frac{D_1(1+g)}{(1+k)^2} + \frac{D_1(1+g)^2}{(1+k)^3} + \dots$$

we can multiply through by  $(1+k)/(1+g)$  to yield

$$\frac{1+k}{1+g}V_0 = \frac{D_1}{1+g} + \frac{D_1}{1+k} + \frac{D_1(1+g)}{(1+k)^2} + \dots$$

and subtracting the first equation from the second, we have

$$\frac{1+k}{1+g}V_0 - V_0 = \frac{D_1}{1+g}$$

implying

$$\frac{(k-g)}{1+g}V_0 = \frac{D_1}{1+g}$$

or

$$V_0 = \frac{D_1}{k-g}$$

<sup>2</sup> For more information on REITs, see <https://www.sec.gov/files/reits.pdf>.

<sup>3</sup> For more information on MLPs, see <https://www.sec.gov/resources-investors/investor-alerts-bulletins/master-limited-partnerships>.

<sup>4</sup> CFA Question answers: 1)B (V=\$39.77), 2)A, 3)C, 4)C, 5)B, 6)C

