University of Richmond
Robins School of Business™

§ 7. Risk and Return

FIN 366: Investments

Table of Contents

Rates of Return

Over One Period

Holding Period Return

The rate of return of an investment to an investor over a single period is known as the **holding period return (HPR)**. It takes into account the change in the price of the investment as well as any income earned, such as a cash dividend. Given a stock price *P* and its dividend *Div*:

$$HPR = \frac{P_{t+1} - P_t + Div}{P_t}$$

The holding period can be for any amount of time (one week, one year, 5 years, etc.). Note that this is the standard percent change formula with an added *income component*.

We can split the HPR into its **capital gains yield** and **dividend yield** components:

$$HPR = \frac{P_{t+1} - P_t + Div}{P_t} = \boxed{\frac{P_{t+1} - P_t}{P_t}} + \boxed{\frac{Div}{P_t}}$$

We assume that the dividend or income component is received at the end of the holding period.

Example: If at the beginning of the quarter, a stock's price is $400, and it falls to $375 at the end of the quarter and then pays a $5 dividend, an investor's HPR for that quarter would be -5%. Their capital gain yield would be -6.25% and the dividend yield would be 1.25%:

$$HPR = \frac{375 - 400 + 5}{400} = \frac{375 - 400}{400} + \frac{5}{400} = -6.25\% + 1.25\% = -5\%$$

Interpretation: The investor's return over the quarter was -5%, including both the change in the stock's price and the dividend it paid.

OVER A YEAR

Investors holding various securities over different time periods may wish to calculate what the returns to each security are on an annualized basis for comparison. The formula to **annualize** returns over different holding periods is

$$Annualized\ Return = (1 + r)^{\frac{Periods\ in\ a\ Year}{Periods\ Held}} - 1$$

where *Periods in a Year* and *Periods Held* are in the same terms (i.e., years, days, quarters, weeks, etc.)

EXAMPLE: An investor's HPRs to various stocks in their portfolio are as follows:

- MA: 6.2% over 100 days
- PFE: 2% over 4 weeks
- HD: 5% over 3 months
- UNH: 29% over 1.5 years (about 548 days)

The *annualized* returns to each are:

$$Annualized\ Return_{MA} = (1 + 0.062)^{\frac{365}{100}} - 1 = 24.55\%$$

$$Annualized\ Return_{PFE} = (1 + 0.02)^{\frac{52}{4}} - 1 = 29.36\%$$

$$Annualized\ Return_{HD} = (1 + 0.05)^{\frac{12}{3}} - 1 = 21.55\%$$

$$Annualized\ Return_{UNH} = (1 + 0.29)^{\frac{1}{1.5}} - 1 \approx (1 + 0.29)^{\frac{365}{548}} - 1 \approx 18.50\%$$

INTERPRETATION: While useful as a point of comparison, it is important that the investor not assume that a return achieved over shorter periods can be maintained for a year. Annualized returns should not necessarily be *forecasts*, especially when annualizing shorter periods.

OVER MULTIPLE PERIODS

Holding Period Returns

Assume now we have an investor's HPRs for a stock or fund over multiple equal-length subperiods. We can determine the HPR over the entire period using this formula:

$$R = (1 + r_1) \times (1 + r_2) \times \cdots \times (1 + r_N) - 1$$

EXAMPLE: If an investor has quarterly HPRs of 3%, -6%, 8%, and 1%, the one-year HPR would be 5.61%:

$$R = (1 + 0.03) \times (1 + -0.06) \times (1 + 0.08) \times (1 + 0.01) - 1 = 0.0561 = 5.61\%$$

INTERPRETATION: The investor's return over the year was 5.61%, which was found using the quarterly returns.

You *cannot* add the returns over multiple periods to find the return over the full period. If you hold a fund for 3 years that returns 10% per year, your HPR is 33.1%, not 30%, due to the power of compounding!

Arithmetic Returns

Over multiple periods, we might want to characterize the HPRs of a stock or mutual fund with the simple **arithmetic average** – adding up the returns and dividing by the number of returns – to give an estimate of the investment's typical return. If your stock returned 4%, 7%, and 5% over three consecutive months, its average return was (0.04 + 0.07 + 0.05) ÷ 3 = 5.33%.

Geometric Returns and CAGR

Investors also use the **geometric average** return for multiple periods:

$$r_{Geometric} = \sqrt[N]{\prod_{i}^{N}(1 + r_i)} - 1 = \left((1 + r_1) \times (1 + r_2) \times \cdots \times (1 + r_N)\right)^{\frac{1}{N}} - 1$$

PRACTICE: A mutual fund has returns of 10%, 20%, -20%, and 20% over the last four quarters. What is the arithmetic and geometric average of quarterly returns over this period?

SOLUTION: The arithmetic average is (0.10 + 0.20 + -0.20 + 0.20) ÷ 4 = 7.5%. The geometric average is:

$$r_{Geometric} = \left((1+0.1)\times(1+0.2)\times(1+-0.2)\times(1+0.2)\right)^{\frac{1}{4}} - 1 = 0.06099$$

$$= 6.099\%$$

INTERPRETATION: The arithmetic average gives us a simple summary measure of what this fund's typical return is in a quarter: 7.5%. The geometric average tells us what the fund *actually* returned per quarter, including compounding and the time value of money. As proof, let's assume an investor invested $100 in this fund 4 years ago:

	Fund Value (Beg.)	Return (r)	Fund Value (End)
Quarter 1	100	10%	100 × (1+0.10) = 110
Quarter 2	110	20%	110 × 1.2 = 132
Quarter 3	132	-20%	132 × (0.8) = 105.6
Quarter 4	105.6	20%	105.6 × 1.2 = **126.72**

The investment grows from $100 to $126.72 using the quarterly returns from the problem or $R = (1+0.1)\times(1+0.20)\times(1+-0.20)\times(1+0.20) - 1 = 26.72\%$.

If instead the investor earned the geometric average = 6.099% every year, the ending value of $126.72 would be the same as above:

	Fund Value (Beg.)	Geo. Avg.	Fund Value (End)
Quarter 1	100	6.099%	106.099
Quarter 2	106.099	6.099%	112.5699
Quarter 3	112.5699	6.099%	119.4356
Quarter 4	119.4356	6.099%	**126.72**

The **arithmetic average** is a useful summary measure of returns that may give a decent estimate of an investment's return *next period*. The **geometric average** is a summary measure that states what the actual average *historical* return was per period, taking into account compounding.

The arithmetic average will always be greater than the geometric average unless the returns are identical in each period. In that case, the arithmetic and geometric average will be the same.

The **compound annual growth rate** (**CAGR**, pronounced *kay-gur*) is the same as the geometric return if you are considering *annual* returns:

$$CAGR = \left(\frac{Ending\ Value}{Beginning\ Value}\right)^{\frac{1}{Years}} - 1$$

If we assume a fund has *annual returns* of 10%, 20%, -20%, and 20%, the CAGR = 6.099%, or:

$$CAGR = \left(\frac{126.72}{100}\right)^{\frac{1}{4}} - 1 = 6.099\%$$

> ⚠ CAGR, by definition, uses *annual* returns. If the returns are over any other period, it is a geometric average. All CAGRs are geometric averages, but not all geometric averages are CAGRs.

Money-Weighted Return

Investors may compute a **money-weighted return** that considers the timing and amount of their investment. Periods where more money is invested will have a larger impact on the overall money-weighted return of the investment.

EXAMPLE: Two investors invest money annually in the S&P 500 index. However, the amount of their investments differ based on the money they have available to invest at the beginning of each year:

	Investor A	Investor B	S&P 500 Return
Time 0	Invests $100 at time 0	Invests $100 at time 0	10%
Time 1	Invests $10 at beginning	Invests $100 at beginning	-10%
Time 2	Invests $200 at beginning	Invests $15 at beginning	18%
Time 3	Cashes out	Cashes out	N/A

At the beginning of time 3, both investors cash out what they've earned: Investor A withdraws $363.44, while Investor B withdraws $240.72.[1] While the arithmetic return is 6% and the geometric return is 5.32% for the fund, the investors' rates of return are different based on the amount and timing of their deposits. Investor A earned 9.699% and Investor B earned 4.812% per year on average. These are their money weighted rates of return, or **internal rates of return (IRR)**.

To find the money-weighted return in the *TI-BAII Plus* calculator:

Keystrokes	**Explanation**
CF	The "cash flow" key, ***CFo=*** appears on the screen
2ND CLR WORK	"Clear worksheet" containing all cash flows (above CE|C key)
CFo = - 100 Enter ↓	Cash flow at time 0: -$100, the initial investment
C01 = - 10 Enter ↓	Cash flow at time 1: -$10, a cash *outflow*
F01 = 1 Enter ↓	Frequency of the $10 cash flow is 1 time
C02 = - 200 Enter ↓	Cash flow at time 2: -$200, a cash *outflow*
F02 = 1 Enter ↓	Frequency of the $200 cash flow is 1 time
C03 = + 363.44 Enter ↓	Cash flow at time 3: $363.44, a cash *inflow* investor withdraws
F03 = 1 Enter	Frequency of the $363.44 cash flow is 1 time
IRR	"IRR" button
CPT	Compute the "IRR" = **9.699%**

Verify that you can get **4.8122%** for Investor B in your calculator.

INTERPRETATION: Although two investors can hold the same fund, the returns they experience can be different based on their investment timing. Investor A timed the market well, investing much of their money before the market rose, while Investor B invested much of their money before the market fell.

Mutual funds and investment companies like Vanguard, BlackRock, and Fidelity often show their account holder's returns using the money-weighted method, given their clients make contributions and/or withdrawals to and from their accounts over long periods of time. This method, however, should not be used to evaluate money managers since they cannot control when investors will give them money to invest.

Arithmetic averages are useful starting points for estimating what a fund will return in future periods. Geometric averages are useful for characterizing the return a fund realized. Money-weighted returns show an investor's return on average given the timing of their investments and withdrawals.

Visit the Excel file *Returns Calculator* available at josephfarizo.com/fin366.html to input your own values and determine returns over various periods.

RISK

Risk considers (1) possible returns and (2) the likelihood such returns are achieved. **Scenario analysis** allows investors to examine returns in different economic and market conditions, which helps to define the tradeoff between risk and return, a foundational concept in finance.

SCENARIO ANALYSIS

PRACTICE: Assume we have an investment in a stock portfolio or mutual fund. We expect there's a 5% chance of a severe recession, 25% chance of a mild recession, 40% chance of normal growth, and a 30% change of an economic boom. In these cases, the portfolio returns -37%, -11%, 14%, and 30%, respectively. Develop a probability distribution of these returns, determine the expected return, and characterize the risk of this investment.

We will use the Excel file *Scenario Analysis* at josephfarizo.com/fin366.html for this example (and to provide additional examples).

SOLUTION: It helps to summarize the information we know in a table. Given probability of a state occurring p(s) and the returns in each state r(s):

Scenario	p(s)	r(s)
Severe Recession		
Mild Recession		
Normal Growth		
Economic Boom		

We first need to find the **expected return** for this portfolio, or the mean value of the distribution of HPRs.

$$E(r) = \sum_{s=1}^{S} p(s)r(s) = p(s_1)r(s_1) + p(s_2)r(s_2) \cdots + p(s_S)r(s_S)$$

That is, we sum the product of the probabilities and returns in each state or scenario:

Scenario	p(s)	r(s)	p(s) × r(s)
Severe Recession			
Mild Recession			
Normal Growth			
Economic Boom			
		E(r) =	

Next, we can observe how the return in each state *deviates* from this expected return we calculated, a step toward quantifying the risk associated with this investment. Each period's deviation from the expected return is the return in each state minus the E(r):

Scenario	p(s)	r(s)	p(s) × r(s)	Deviation
Severe Recession				
Mild Recession				
Normal Growth				
Economic Boom				
		E(r) =		

Now, we can compute the **variance** of this portfolio:

$$Var(r) = \sigma^2 = \sum_{s=1}^{S} p(s)[r(s) - E(r)]^2$$

By the formula, the variance requires:
(1) Squaring the deviations
(2) Multiplying each by its probability
(3) Summing all *squared deviations*

Squaring the deviations prevents our negative deviations from "offsetting" our positive deviations.

Scenario	p(s)	r(s)	p(s) × r(s)	Dev.	$Dev.^2$	p(s) × $Dev.^2$
Severe Recession						
Mild Recession						
Normal Growth						
Economic Boom						
		E(r) =			Var =	

The variance is difficult to interpret. A greater variance implies greater risk, but it is in "squared units". Converting to the **standard deviation** converts the risk measure to the same units as the returns (that is, in percent).

$$SD(r) = \sigma = \sqrt{Var(r)}$$

Thus, our standard deviation is:

$$SD(r) = \sigma = \sqrt{Var(r)} = \sqrt{\qquad\qquad} =$$

Recalling that for normally distributed returns, *approximately*:

- 68% of observations ± 1 standard deviation from the expected value.
- 96% of observations ± 2 standard deviations from the expected value.
- 99% of observations ± 3 standard deviations from the expected value.

INTERPRETATION: Given returns in different states of the world and their probabilities, we can determine an expectation for the return on a portfolio. We can quantify the risk as well, using a rule-of-thumb for standard deviation to determine the likelihood we achieve some return in the future.

Our results indicate that we expect this portfolio to have returns between:

_____________ and _____________ 68% of the time.

_____________ and _____________ 96% of the time.

_____________ and _____________ 99% of the time.

We used annual returns in this problem, so we assume *annual* returns in our estimates. The "percent of the time" would be "percent of *years*."

Returns greater than _____________ or less than _____________ are expected to be rare, occurring less than 1% of the time.

Analysts select probabilities based on research and forecasts. In the previous example, perhaps the "probability of severe recession" of 5% is because in the analyst's opinion, 1 of the previous 20 years was a severe recession, and the portfolio returned -37% in that year. In their opinion, 6 of the previous 20 years (30%) were economics booms, and the arithmetic average return of the portfolio was 30% in those years.

THE HISTORICAL RECORD

Often, analysts use the historical record of a portfolio's returns to determine the expected returns and standard deviation. A simple arithmetic average may serve as the expected return, while the standard deviation of a sample of returns is:

$$SD(x) = \sigma = \sqrt{Var(x)} = \sqrt{\frac{\sum(x-\bar{x})^2}{N-1}}$$

where x is a period's return, and $\bar{x}$ is the average return of N periods.

EXAMPLE: We can determine an expected return and standard deviation for the market, using real-world financial data of the S&P 500 index. The returns over the last 20 years are in the table below.

Year	S&P 500 Return		Year	S&P 500 Return
2023	26.06%		2013	32.15%
2022	-18.04%		2012	15.89%
2021	28.47%		2011	2.10%
2020	18.02%		2010	14.82%
2019	31.21%		2009	25.94%
2018	-4.23%		2008	-36.55%
2017	21.61%		2007	5.48%
2016	11.77%		2006	15.61%
2015	1.38%		2005	4.83%
2014	13.52%		2004	10.74%

The arithmetic average is 11.04%. We can use this as an expected return, and the $\bar{x}$ in the standard deviation equation. The standard deviation would be:

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N-1}}$$

$$\sigma = \sqrt{\frac{(0.2606-0.1104)^2+(-0.1804-0.1104)^2+\cdots+(0.1074-0.1104)^2}{20-1}}$$

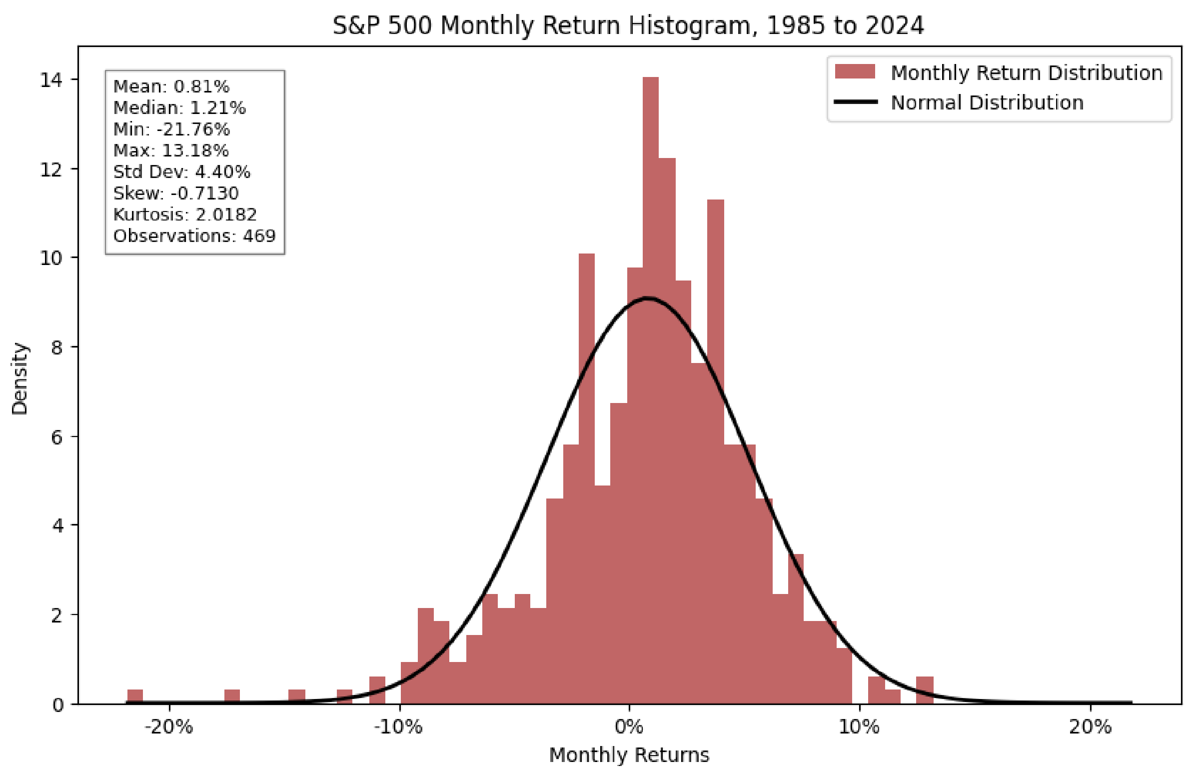
$$\sigma = 0.1688 = 16.88\%$$

INTERPRETATION: Based on the last 20 years of data, an analyst might conclude the S&P 500 has an expected return of 11.04% with a standard deviation of 16.884%. Thus, we expect S&P 500 returns to be:

- Between 11.04% ± 1(16.88%) = [-5.84%, 27.92%] in 68% of years
- Between 11.04% ± 2(16.88%) = [-22.72%, 44.8%] in 96% of years
- Between 11.04% ± 3(16.88%) = [-39.6%, 61.68%] in 99% of years

Stock and asset returns do not perfectly follow normal distributions. Returns may be **skewed**, rather than symmetric. **Negative skew** implies extreme negative values have been observed, while **positive skew** implies extreme positive values have been observed. Return distributions may exhibit **kurtosis**, or a greater probability that extreme positive or extreme negative returns are likely to occur than predicted by the normal distribution.

Figure 1: Comparing Stock Returns to Normal Distribution



You can generate the most up-to-date histogram of S&P 500 monthly returns by running the code *S&P 500 Return Distribution* at josephfarizo.com/fin366.html.

Risk and Return Tradeoff

With a clearer understanding or rates of return and risk, we can consider them in tandem. Investors must consider how much of an expected reward is offered to compensate for the risk of an investment. Expected returns alone do not disclose riskiness. We look at two measures that consider both returns and risk: (1) the **coefficient of variation** and (2) the **Sharpe ratio**.

Coefficient of Variation

The **coefficient of variation** (**CV**) is the ratio of the standard deviation to the mean (or expected value).

$$CV = \frac{\sigma}{\bar{x}}$$

The CV allows for comparison of risk or dispersion across different investments. A stock with a higher CV than another stock, for example, implies that the stock has greater risk relative to the returns it offers.

Sharpe Ratio

The **Sharpe ratio** gives us the reward per "unit" of risk. The higher the Sharpe ratio, the more reward an investment offers over the risk-free rate given its level of risk.

$$S = \frac{E(r_P) - r_f}{\sigma_P}$$

The numerator $E(r_P) - r_f$ is the **risk premium** for the portfolio P: the expected return minus the risk-free rate. The denominator is the standard deviation for the portfolio P. In practice, the actual realized return may be used in the numerator, implying the formula uses the **excess return** in the computation. This may be a reasonable approach, given realized returns may provide reasonable forecasts for an expected return.

- **Risk Premium**: an expected return in excess of the return on risk-free securities, such as the 90-day US Treasuries (T-bills); the reward.

- **Excess Return**: the actual return in excess of the return on risk-free securities.

- **Risk Aversion**: the reluctance to accept risk, implying that investors demand a higher reward to accept higher risk.

The Sharpe Ratio should be used when comparing diversified portfolios of securities, rather than individual securities. As we will see later, we have a different measure of risk for individual securities, where we will need to account for their correlations.

EXAMPLE: A technology sector mutual fund has an expected return of 10% with a standard deviation of 18.6%. An energy sector mutual fund has an expected return of 13% and a standard deviation of 28%. Assuming a risk-free rate of 2%:

$$S_{tech} = \frac{E(r_{tech}) - r_f}{\sigma_{tech}} = \frac{0.10 - 0.02}{0.186} = 0.4301$$

$$CV_{tech} = \frac{\sigma_{tech}}{\bar{x}_{tech}} = \frac{0.186}{0.10} = 1.86$$

$$S_{energy} = \frac{E\left(r_{energy}\right) - r_f}{\sigma_{energy}} = \frac{0.13 - 0.02}{0.28} = 0.3928$$

$$CV_{energy} = \frac{\sigma_{energy}}{\bar{x}_{energy}} = \frac{0.28}{0.13} = 2.15$$

INTERPRETATION: The tech fund's Sharpe is higher and its CV is lower than the energy sector, implying it is the preferable investment. The tech fund's return better compensates for its risk than the energy sector. Nevertheless, investors should keep in mind that past returns are not indicative of future returns, and expected returns are only estimates.

Critical Thinking Questions

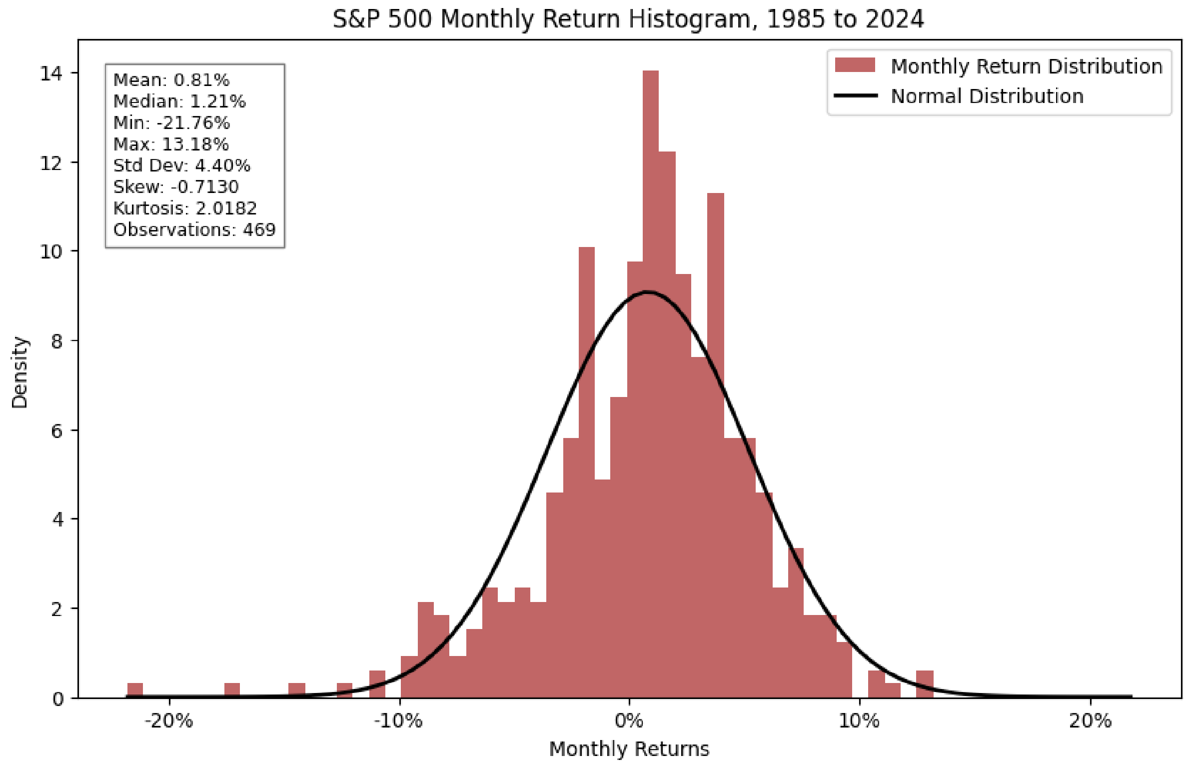
1. When should an investor consider computing and using arithmetic returns? When should an investor consider computing and using geometric returns?
2. A stock's 2 day holding period return is 5%. What is its annualized return? Do you think this return will be realized?
3. A friend tells you "The arithmetic average of a fund's annual returns over the last 20 years is 12%. Therefore, an investor who invested 20 years ago in this fund earned 12% per year." Critique this claim.
4. A friend tells you "The geometric average of a fund's annual returns over the last 20 years is 12%. Therefore, a reasonable forecast for this fund's next period return is 12%." Critique this claim.
5. A friend tells you "The geometric average of a fund's annual returns over the last 20 years is 12%. Therefore, an investor who invested 20 years ago in this fund earned 12% per year." Critique this claim.
6. Explain why two investors that both invest in the same mutual fund over the same time period can have different average rates of return. What calculation would you use to measure their average rates of return?
7. Explain why active mutual fund managers' performance should not be judged based on money-weighted rates of return.
8. How can an analyst find or develop the probabilities of economic states and portfolio returns in each state when conducting scenario analysis?
9. What is the interpretation of the standard deviation and what are its units?
10. Can a security's standard deviation be greater than its expected return? Why or why not?
11. Can a security's standard deviation be negative? Why or why not?
12. What is the difference between a risk premium and excess return?
13. Is a portfolio with a 30% expected return "better" than a portfolio with a 20% expected return?
14. How do returns distributions differing from normal distributions change our perceptions of risk? What might negative skewness and high kurtosis mean for the likelihood of negative or extreme events?
15. Is a portfolio with a standard deviation of 8% "better" than a portfolio with a standard deviation of 14%?
16. If a portfolio has a better Sharpe ratio than another portfolio, will it also have a better CV than that other portfolio?
17. If a Sharpe ratio uses the risk premium in the numerator, will it ever be negative? Can the Sharpe ratio be negative if it uses the excess return in the numerator?
18. Challenge Over the past 100 years, the monthly return of the stock market overall has been about 0.67%, with a standard deviation of about 5.34%. Given our interpretation of standard deviations and the 68 %, 95%, 99% rule, about how many months out of the 1200 (12 months × 100 years) would we expect the market to return less than -15.35%? How many months out of 1200 would we expect the S&P 500 to return +16.69%? In reality, 0.87% of months have returns below -15.35% and 0.52% of months with returns greater than 16.69%. What does this

imply about the distribution of stock returns? Are they normally distributed? What does this mean for forecasting *next* period's return?

19. CHALLENGE **Black swan** events are highly unlikely and highly impactful events in markets. The name is derived from a story: a person who has only observed white swans at a local pond over 100 years of their life might conclude that black swans do not exist (while they certainly do). What lesson can we take away from the existence of black swans in financial markets? Do we think extreme events are predicted by assumptions of normal distributions? How does this relate to kurtosis?

ANALYTICAL QUESTIONS

1. Using the figure below, answer the questions that follow.



a. What is the average, median, and standard deviation of monthly returns of the S&P 500 over the given period?
b. What is the number of observations, and what does that mean? What do the vertical bars represent? What does the curved line represent?
c. *If* returns followed a normal distribution, what would the range of returns be for 1, 2, and 3 standard deviations following the 68%, 96%, and 99% rule?
d. The figure indicates negative skewness and a mean below the median. How do the minimum and maximum returns on the figure help to explain this negative skewness?
e. Given your answers to the questions above, how might you caution investors when interpreting the 68, 96, 99 standard deviation rule?

CFA Questions

Answers are in the *Notes & References* section below.[2]

Below are the assets under management and returns of the Valley Superior Fund. Note that assets under management takes into account both returns of the portfolio and customers adding or withdrawing from the fund, which is why, for example, the 45 million in year 2 is not 15% greater than the 30 million in year 1. Use this information to answer the questions 1 through 4.

Year	Assets under Management at the Beginning of Year (euros)	Annual Return (%)
1	30 million	15
2	45 million	−5
3	20 million	10
4	25 million	15
5	35 million	3

1. The fund's arithmetic mean annual return is:
   a. 0.0667%
   b. 7.60%
   c. 0.0760%

2. The fund's geometric mean annual return is:
   a. 7.32%
   b. 8.11%
   c. 1.0732%

3. The fund's holding period return over the five-year holding period is:
   a. 42.35%
   b. 16.67%
   c. 0.1667%

4. To evaluate the Valley Superior Fund manager's performance, we should use the
   a. Money-weighted return
   b. IRR
   c. Geometric average

5. Determine the investor's money-weighted return, given they made the following series of investments over 5 years, and cashed out at the end (*The proper keystrokes are in the answer key. Attempt on your own first, paying close attention to Time 2's cash flow*):

	Investor A
Time 0	Invests $30 at beginning
Time 1	Invests $10.50
Time 2	Withdraws $22.75
Time 3	Invests $3
Time 4	Invests $6.25
Time 5	Withdraws balance: $36.05

   a. 6.618%
   b. 6.703%
   c. 5.855%

6. A fund receives investments at the beginning of each year and generates returns for three years as follows:

Year of Investment	**Assets under Management at the Beginning of each year**	**Return during Year of Investment**
1	USD1,000	15%
2	USD4,000	14%
3	USD45,000	−4%

   Which return measure over the three-year period is negative?

   a. Geometric mean return
   b. Arithmetic mean return
   c. Money-weighted return

7. Which ETF has the highest annualized rate of return?

ETF	Time Since Inception	Return Since Inception (%)
1	125 days	4.25
2	8 weeks	1.95
3	16 months	17.18

a. ETF 1
b. ETF 2
c. ETF 3

NOTES & REFERENCES

[1] To determine how much each investor has available to withdraw at the end, sum up the amount they invest/withdraw in each year with the rate of return they earned in that year. I show where Investor A's numbers come from by matching colors across different columns:

	Investor A Cash Flow	Investor B Cash Flow	S&P 500 Return	Investor A's Balance	Investor B's Balance
Time 0	Invests \$100	Invests \$100	10%	\$100 grown at 10% = \$110	\$100 grown at 10% = \$110
Time 1	Adds \$10	Adds \$100	-10%	(\$110 + \$10) grown at -10% = \$108	(\$110 + \$100) grown at -10% = \$189
Time 2	Adds \$200	Adds \$15	18%	(\$108 + \$200) grown at 18% = **\$363.44**	(\$189 + \$15) grown at 18% = **\$240.72**

[2] CFA Question answers: 1) B, 2) A, 3) A , 4) C, 5) C. *The keystrokes are CFo = -30, C01 = -10.50, F01 = 1, C02 = +22.75, F02 = 1, C03 = -3, F03 = 1, C04 = -6.25, F04 = 1, C05 = +36.05, F05 = 1, IRR, CPT = 5.855%*, 6) C, 7) B